Learning the Basic Facts:
Opportunities to develop important algebraic understandings

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Effect of Scaffolded Instruction in the Context of Basic Fact Strategy Use

(A)
Traditional Instructional Approach

Instructional Zone ←→ memorized facts

In the absence of instructional support
Revert to Comfort Level

Comfort Zone ←→ counting on fingers by ones

(B)
Algebraic Lens Instructional Approach

Instructional Zone ←→ Number Recall

In the absence of instructional support
Revert to Comfort Level

Comfort Zone ←→ Derived & Flexible Choice Strategies

Now muted, diminished strategy ←→ Counting on fingers by ones

Figure One

Learning the basic facts is a rite of passage for elementary children. How fast a child recalls facts can be, rightly or wrongly, a gatekeeper to more advanced mathematics classes. There is no question that being fluid with the basic facts increases the ability to work through more advanced problems. However, it has also been proven that a child can be quite sophisticated at problem solving before he or she has any basic facts committed to memory. The knowledge of these facts grows through a child's focused problem-solving activities beginning with simple combinations and increasing to include a growing repertory of other combinations (Carpenter, Fennema, Franke, Levi, & Empson, 1999).

Many teachers become frustrated with students and their inability to commit these facts to memory. One typical complaint is reliance by some children to count on their fingers to compute answers. Counting on is an early efficiency children naturally discovered as they work with numbers. Their fingers are readily available as manipulatives. Culturally in the United States, however, finger counting is frowned upon. But how do we assist children in moving beyond the counting by ones strategy to something more efficient? In their research on how children develop mathematical ideas, Carpenter, et. al. supported evidence that there is a stage of development between counting on and fact recall. This is the stage of deriving and flexible choice of strategies.

Why the Persistent Use of Fingers

Developmental learning theory has outlined how students, when not scaffolded at their instructional level, revert to their "comfort zone" for their independent level (Vygotsky, 1934/1986). Literacy teachers who use Guided Reading follow this model in planning instruction for their students. The same model can be used to understand why students persist with counting on their fingers. Students develop early the capacity to *count on* by ones using their fingers to keep track of their mental actions. While some curriculum resources may spend some time on strategy development such as counting on form the larger, doubles plus or minus one, or making a ten, this support is typically perfunctory. The general expectation is that the student, with the support of their families, will commit these basic fact combinations to memory. The outcome of this approach is, if a child doesn't remember 8 + 5 *yet*, he or she will revert to their comfort zone – counting on by ones – to calculate the answer. Children who have not been instructionally supported in deriving and flexible choice of strategies for a *sustained period of time* will revert to counting on by ones when they can't recall a fact combination because it is the only strategy choice that they have. (See *Figure One*.)

Liping Ma (1999) notes a difference in approach between American and Chinese teachers in the development of the basic facts such as 5 + 7 or 12 – 7. American teachers typically see "basic arithmetic facts as items to be memorized." The Chinese, however, consider such problems as "addition with composing and subtraction with decomposing within 20" (p. 16). What does this mean? What do the terms composing and decomposing indicate mathematically? How does deriving and flexible choice of strategies aid, not only the recall of facts, but also the development of important higher mathematical ideas?

Looking at the basic facts through the lens of key algebraic concepts reorients one from the notion that the basic addition, subtraction, multiplication, and division combinations are a set of items to be memorized to an opportunity to explore and develop an important set of interconnected mathematical ideas. These mathematical ideas, if learned with small numbers, can make working with larger numbers easier and makes explicit key algebraic properties of number operations. This is the argument for why sustained instructional support for learning derived strategies around the basic facts requires a place in classroom instruction.

Arithmetic to Algebra – Big Ideas Behind the Basic Facts

Take a copy of an addition time test and display it visually for the whole class to see. Ask your students which are the combinations that they know right away without thinking. Ask the students why they are so easy. Typically, one of the first items they will identify is a number plus zero, e.g., 6 + 0. Without much prompting most first graders will express the identity property of addition by saying, "any number plus zero gives you that number." The same is true for 0 + 6; "Zero plus a number gets you the number you added."

The students will also identify a "number plus one", e.g., 5 + 1, as easy because it's "just the next number up." The doubles such as 2 + 2 and 5 + 5 also are recalled easily as known combinations. These basic statements are conjectures that young children develop as they work with numbers. Tapping into these intuitive conjectures and making them explicit in the classroom is one important avenue a teacher can take in developing deep understanding of the mathematics that lies behind these basic facts (Carpenter, Franke, & Levi , 2003). The concept of zero is multifaceted. The empty set, e.g., "zero is nothing," is only one context of zero. Exploring number concepts such as this are highly engaging to children. The same is also true with the identity property in multiplication.

Now ask the students to identify which ones are the hard ones. Which ones do they find themselves having to do some counting? For addition, combinations such as 8 + 6 or 9 + 7 are often identified. For multiplication, 6 x 7 or 8 x 6 are common. The question to then pose to the students, *Other than counting on your fingers, what other strategy can we use? Is there a way to break the numbers apart so we can use an easier combination to figure out the harder combination?* These kinds of open but focused discussions among the students with the teacher providing mathematical representations to make the mathematics visible lead to explicit conversations around key algebraic ideas. It is up to the teacher to nurture these connections.

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The Commutative Property

One of the earliest efficiencies a child discovers when working with number is that he or she can hold one number mentally and count on the second set. Eventually the child discovers that counting on from the larger number is even more efficient. In the process of doing this, he or she intuitively discovers the commutative property.

The commutative property (a + b = b + a and a x b = b x a) opens up other possibilities for students to explore that reduces the number of facts they need to learn. Having students explore the range and limits of this algebraic principle, when it works (addition and multiplication) and when it does not (subtraction and division), allows the students to develop key insights into how the four number operations work individually and in relation to each other. Again, many students can only take advantage of this principle if the ideas behind the commutative property are specifically explored and developed. Some may internally develop the ideas, but many in the class will leave the principle unconnected and underdeveloped.

Decomposition of Number

A number is composed of subsets of other units. This is a foundational idea for operating on any number. If working with the set of whole numbers, the number 7, for example, can be broken into the combined values of 5 + 2, 4 + 3, 6 + 1, among others. The process of breaking numbers into subsets is *known as decomposition of number*. Deriving strategies are built on this concept. To think flexibly about number in this manner is critical to a student's mathematical development. Many children who struggle in mathematics have not developed this concept. They are limited to seeing and working with number as a single unit or as a collection of ones. The persistence of counting all numbers by ones may be an indication that the capacity to see numbers in decomposed sub-combinations has not been developed by the student.

Working with dot cards and ten frames are a visual means of assisting students in developing the capacity for *conceptual subitizing* (Clements, 1999). This is the ability to look at any combination of dots and determine the total quantity by relating its parts to the whole. This is the same conceptual idea of making units of units in multiplication. Look at *Figure Two*. Notice how your eye determines the total quantity. Flashing such a card to a child for them to quantify the number of dots, but not so long that the dots are counted one by one. The conversation around how many did you see and how did you see leads to the various ways the eye grabbed and combined the information. Did you see it as 4 + 3, 3 + 3 + 1…? The importance is that there is no one way to determine the total quantity, but that the eye worked with chunks of the number rather than as a single unit.

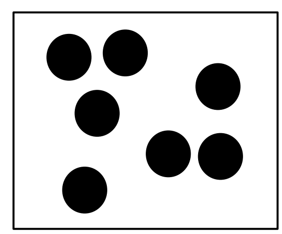
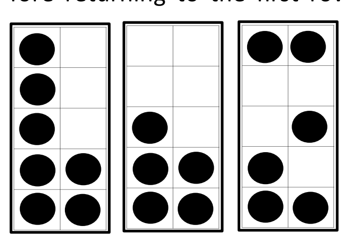


Figure Two

The same idea can be presented with the use of ten frames. There are three structures of the ten frames that support visual images of a number as a subset of other numbers. The "five-wise" pattern completely fills the one row of five before adding dots to the second row. "Pairwise" frames fill in one in the first row, the second in the second row before returning to the first row. These pairings are useful in supporting conversations around why a number is odd or even. The third arrangement is random where sub-clusters of dots need to be combined to determine the total. Figure three shows examples of each. Notice how the eye has to work to determine the total within the frame of ten.



(A) Five-wise (B) Pair-wise (C) Random

Ten Frame Patterns

Figure Three

The decomposition of number is the core idea behind all deriving strategies. Intentionally developing this strategy as a bridge between counting on by one strategies and recalling the known fact is essential. One reason children revert to counting on their fingers when they can't remember a fact is because they have not developed the interim strategies involving deriving. Children using derived facts see the basic facts as in interrelated network of associated combinations instead of isolated individual factoids. Their long term memory is an integrated network of ideas that eventually allows them to recall facts more effectively.

The following first looks at strategies related to addition and subtraction strategies followed with a look at multiplication and division.

Addition – Derived Strategies

Doubling, Plus or Minus

Children, very early on, develop a fascination with the concept of doubling. 1 + 1, 2 + 2, 3 + 3, 5 + 5, 10 + 10 are combinations that are easily committed to memory. They are fascinated by the patterns, and more adept students take on the challenge of learning higher double combinations. Building off doubles to learn other facts reflects a significant understanding about relations among numbers.

Take the basic fact 5 + 6. Those who don't know the fact automatically, and cannot break individual numbers into subsets, would need to count by ones to solve the problem either through direct modeling – counting *both* numbers in sequence – or counting on from one of the two numbers. A child who can decompose numbers uses information about 5 + 5 or 6 + 6 to solve the problem. If the child decides to use 5 + 5 it is because they understand that 6 = 5 + 1 so therefore 5 + 6 = 5 + 5 + 1. The decomposing of the 6 into 5 + 1 requires, at least, an intuitive sense of equality – another algebraic concept. If the child chooses 6 + 6 as a starting reference, he or she has to deal with a fact that the relationship with the original fact is unequal ($5 + 6 \neq 6 + 6$). This requires some thought as to how to adjust the equation to reestablish equality. Since 6 + 6 is one larger that 5 + 6, the child subtracts one number to regain a state of equality (5 + 6 = 6 + 6 – 1). The relational thinking developed requires an understanding of how numbers can be decomposed into useful combinations, re-associated, and kept equal.

Make a Ten

As children explore the base ten system, they come to realize that ten plus a single digit number combines to make those two numbers one number, e.g., 10 + 3 = 13. While hard for many to articulate, the implicit algebraic conjecture that makes any single digit easy to combine with a group of ten is "$0 + a = a$" The language of the teens makes it difficult for students to comprehend this place value concept. Many young children write 71 for seventeen as they hear the "7" first and the "teen" last. Seeing the teens as tens and ones, rather than a collection of ones, sets the stage for why reconfiguring numbers around a ten is a significant efficiency. Getting to a ten makes adding the remaining single digits very quick and efficient. A basic fact such as 7 + 4 sets the stage for a child to compose a ten. The problem joins two single-digit numbers to a ten plus a single-digit number. Decomposing the seven or the four to then re-associate one of the addends with the other allows the child to compose the ten, then add the single digit to the ten to make the final composite number. Example:

7 + 4 = 7 + (3 + 1) *[decomposition]*
= (7 + 3) + 1 *[associative property]*
= 10 + 1

9 + 4 = 9 + 1 + 3 *[decomposition]*
= (9 + 1) + 3 *[associative property]*
= 10 + 3

Viewing this decomposition and re-association process via the ten frame cards is useful for those students still becoming secure with this next progression in their thinking. Determining the total number of dots, without counting them by ones, engages the eye in a manner that can be directly linked to the numeric decomposition and associations shown above. Helping students see these connections strengthens the mathematical concepts and scaffolds them off the need to count all numbers by ones. A crucial aspect of using any representational tools such as these, is the conversations orchestrated by the teacher in asking strategic questions of what the student sees, how it is seen, and how it is the same of different from another student's strategy that the mathematical ideas become fully formed and numerically represented. Otherwise, such activities become perfunctory procedures rarely accessed by the student in their everyday problem solving.

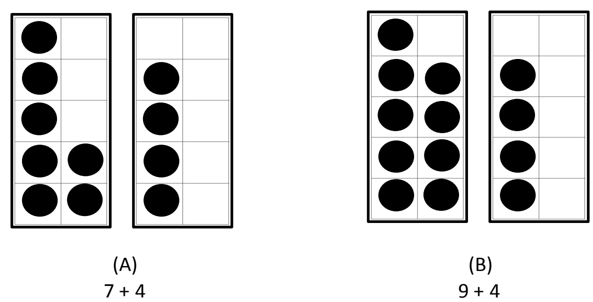


Figure Four

Researchers have identified fluidly knowing the ten facts as key to working with multidigit numbers. In using that very quick mental reconstruction, the child decomposed a number, configured a ten, and used the associative property and concepts of equality.

Eventually one expects a student to know the facts. However, if a student does not automatically know facts such as 8 + 5 or 7 + 9, he or she can potentially solve the facts fluidly by decomposing and reconfiguring numbers. The explicit public conversations around the underlying mathematical ideas allow the algebraic and relational thinking processes to emerge. Fluidity with these strategies with smaller combinations makes working with multidigit numbers much easier.

Subtraction – Derived and Flexible Choice Strategies

Get Back to a Ten Strategy

The earliest conception of subtraction for children is "take away." Take away is based on the concept of "before," e.g., 5 – 1. Without this concept, a child is hampered in solving problems involving separating a number from another. Counting back by ones is the initial solution strategy students use. In order to move beyond counting off by ones, a student needs to decompose numbers to move backwards in larger chunks. Consider 13 – 5. If the child does not automatically know the fact, decomposing the 5 into 3 + 2 enables the quick removal of the three to get back to the ten. Thus 13 – 3 → 10 – 2 → 8 allows the child to move under the ten efficiently.

To execute this strategy, however, two key concepts need to be understood; that 13 = 10 + 3 and the ten fact of 8 + 2 = 10. Another efficiency is perceiving that, while 5 = 4 + 1 and 5 = 3 + 2, 3 + 2 is selected in order to eliminate the 3 from the 13 to get 10. Having a child constantly verbalize this decision-making process aids in understanding the efficiency.

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The Difference Between Two Numbers

While "take away" is the most common construct of subtraction, it is not the only one. The "difference between" two numbers is another conceptualization of the operation that in many instances is more efficient. Consider 13 – 8. Imagine that you can only solve this with counting back by ones on your fingers. Starting verbally at 13, and raising a finger beginning with 12, then 11, 10, 9, 8, 7, 6, 5 so that eight fingers are raised, you determine the answer is five. That solution follows the structure of 13 – 8 = *x.* Knowing that subtraction can also be perceived as the *difference between* two numbers restructures the task as 13 – *x* = 8. Counting back by ones this way, only five fingers are needed. Given the two numbers of 13 and 8, taking away eight is more tedious, less efficient, than perceiving the task as the difference between the two numbers. However, when the numbers are 13 and 5, taking away five is less work, i.e., more efficient, than counting back *to* five. The number relations determine which conceptualization of subtraction is the most efficient.

Using the difference between concept often triggers the inverse operation. Instead of subtracting, the individual transforms the problem into a join, change unknown task and solves by counting on. For instance, 13 – 8 = *x* is transformed into 8 + *x* = 13. Understanding the inverse relationship of addition and subtraction simplifies learning subtraction facts. Adding is easier than subtraction, thus restructuring into 8 + *x* = 13 is simpler for some. Understanding the inverse operation, however, requires an abstract flexibility that is sophisticated, and takes time to develop. Once developed, a child initially counts on by ones to solve the problem. The long term efficiency is to add 2 to get to a 10 then add 3 to get to 13 (8 + 2 → 10 + 3 → 13, the answer is 5).

The "difference between" and "inverse operation" strategies arise if problems generating such situations are presented to the students. The compare, difference unknown and both the join and separate change unknown problem types are well suited to foster the inverse relations between the operations of addition and subtraction. The concept of the inverse may arise on its own, but more likely if the teacher presents problems, games, and discussions that intentionally explores these ideas.

Multiplication

Up to now, discussion has been centered on addition and subtraction, but similar relationships can be explored within multiplication. What is multiplication and how is it different than addition? There are multiple conceptions of the operation but a typical entry point for most students is the equal grouping construct of multiplication. 3 + 4 is obviously different than 3 x 4. More importantly, the 3 in each expression means something fundamentally different. The '3' in the addition expression indicates three individual items. The '3' in the multiplication expression means three equal groups of a certain number of items within each group.[1]

Multiplication is formed around the concept of making units of units to form new composite units that can then be counted. However, with each repetition of the new composite unit, a simultaneous increase in the number of subunits also occurs. Thus multiplication requires the coordination of units within all sets. Furthermore, in multiplication the units transform as the product is determined. 3 bags of bagels with 4 bagels per bag results in 12 bagels. The "bags" disappear as a unit in the description of the 12 bagels. Part of learning the basic facts in multiplication requires attention to the meaning of the operation itself and the units that are ascribed to the various factors and products. Thus an exploration of the basic facts in multiplication – and subsequently division – is an exploration of the operation itself.

Students' initial strategy to multiplication facts is through the additive approach of repeated addition. This leads to skip counting which students find useful. However, as with counting on by ones in addition, the goal is to move students off additive and skip counting strategies to more multiplicative based ones. The mathematical concept of decomposition of number is key.

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The Distributive Property of Multiplication of Addition

If the student does not know 7 x 6 *yet*, but knows 7 x 5, he or she can use their understanding of how multiplication works to connect 7 x 6 to 7 x (5 + 1). This creates an image of

[1] There are two conventions about which factor is the multiplier (number of groups or scale factor) and which the multiplicand (number in each group). The convention of 3 groups of 4, the three is the multiplier and is the convention most commonly used in America. The phrase "3 times 4," meaning "I want 3, 4 times, 4 is the multiplier. This convention is most common in Europe and Asia. Whichever convention is used, it is important to define in conversation which convention is being used so that common meanings are understood. For this article, the multiplier is always first.

divvying up the items within the groups into subsets of 5 and 1. One can then calculate 7 groups of 5 plus 7 groups of 1. The combination of those partial elements creates the total product. It is also possible that the multiplier could be broken up into addends so that a familiar number of sub-groups can be worked with. In the case of 7 x 8 (7 groups of 8), if one knows that 7 = 5 + 2, then 5 groups of 8 plus another 2 groups of 8 is another means to determine the total of 7 x 8.

What students begin to do intuitively, and that needs to be drawn out explicitly by the teacher, is the distributive property.[2] At its core, the distributive property is possible by decomposing one or both factors into addend components, multiplying the partials, and then recombining them to find the total product. Teachers can nurture this understanding by capturing student intuitive strategies as well as through the use of open number sentences to explore and confirm these relationships. Example:

$8 \times 6 = (4 + c) \times 6$

$8 \times 6 = 8 \times 3 + 8 \times c$

$c \times 6 = 5 \times 6 + 3 \times 6$

Two overriding messages are conveyed to students as these decompositions are explored. First, *use what you know about easier problems to figure out the harder problems* rather than skip counting everything. Second, *if you do not like the numbers as they are, break them apart to make them easier to work with.* These are the core ideas around deriving.

Distributive Property and Place Value

The facts involving a teen allow students to use place value understanding to solve problems. 9 x 12 is easily solved as 90 + 18 if the child understands that 12 = 10 + 2. Once again, the distributive property is intuitively used by the child: 9 x 12 = 9 x (10 + 2) = (9 x 10) + (9 x 2).

Factoring and the Associative Property

Why does 4 x 6 have the same answer as 2 x 12? Is there a way to break the numbers apart to prove mathematically why? Posing this questions draws out another decomposition strategy. Instead of breaking the numbers into addends, 4 or 12 can be factored into 2 x 2 or 2 x 6, respectively. Thus…

$$\begin{aligned} 4 \times 6 &= 2 \times 12 \\ &= 2 \times (2 \times 6) \quad \textit{[decomposition]} \\ &= (2 \times 2) \times 6 \quad \textit{[associative property]} \\ 4 \times 6 &= 4 \times 6 \quad \textit{[reflexive property]} \end{aligned}$$

While students' initial attempts at this arise through doubling and halving, combinations such as 9 x 7 can be factored into (3 x 3) x 7 to form an easier combination of 3 x (3 x 7). In both, the number is decomposed into factors and then the factors re-associated to form easier combinations. The algebraic associative property can be explored and discussed. The combination used depends upon the bank of known facts within a particular student's repertory.

[2] It is not the vocabulary phrase, distributive property, that need be the focus of the conversation, but rather the act of decomposing the numbers into addends, multiplying the partials and then combining those to find the total that should be the initial focus of conversation. Attaching the title of that process as in, *the term mathematicians use for this is the distributive property,* may be appropriate at fourth or fifth grade but not a necessity for second and third graders as they first engage with the strategy.

Division

As with multiplication and addition, helping students comprehend the interrelationships between division and subtraction is important. 12 ÷ 3 can be solved using repeated subtraction: 12 – 3 → 9 – 3 → 6 – 3 → 3 – 3 → 0, the answer is 4 groups of 3. This is using a measurement division conceptualization of the operation: how many threes are in twelve ($a \times 3 = 12$)?

An alternative conceptualization is partitive division: how many in each set if each set gets the same amount? Or share 12 with 3 equally ($3 \times a = 12$). To some this organization makes more sense. 12 ÷ 3 using this conceptualization uses established number sense and known facts to solve the problem. I know 2 items in each set would use up 6 of the 12 (3 x 2 = 6). That leaves another 6 to pass out so another 2 items per group for a total of 4.

As students first begin to develop a sense of division, they need problem solving contexts that allow them to explore both the measurement and partitive structures. A child can approach a problem such as 48 ÷ 8 by sorting through a range of strategies to find the most efficient. With meaningful explorations, guided conversations to compare strategies, and practice, a student will move towards more abstract, more efficient strategies.

Decomposing 48 ÷ 8 into (40 + 8) ÷ 8 = 40 ÷ 8 + 8 ÷ 8 is one way to make the problem easier. A child may not know 48 ÷ 8 but knows 40 ÷ 8 and 8 ÷ 8. Just as easily, a child can decompose the problem into 48 ÷ (2 x 4) = 48 ÷ 2 ÷ 4. Or even 48 ÷ 8 = 24 ÷ 4 = 12 ÷ 2 if those relationships are understood and explored as to why each results in the same quotient. The order of operations becomes an explicit conversation with student as these strategies are explored, enriching the mathematical conversations even further.

Knowing the inverse relationship between multiplication and division is how many students talk themselves through the basic facts. "What times 8 equals 48?" ($c \times 8 = 48$) is a frequent phrase as is "8 times what equals 48? ($8 \times c = 48$). Drawing students' attention to these relationships will empower them to utilize the strategies when an automatic answer fails them. In the process, they broaden the network of related important mathematical principles with which to solve problems with accuracy.

Why Bother

To many of us, all of this seems like too many steps. Just memorizing the fact itself would be much simpler. However, having children use these strategies actively while building their fact knowledge will establish a network of associated ideas in the brain that explores rich and comprehensive mathematical concepts. These mathematical concepts are, in fact, essential for working with multidigit numbers and form the basis for more advanced work in algebra.

Flash Cards & Timed Tests

There is a place for memorization, repeated practice, and timed tests. The question is when. Research shows that problem-solving skills, while enhanced by knowledge of the basic facts, is not dependent upon them. Children can be excellent problem-solvers and not be fast on a timed test. Knowledge of facts should not be a gateway to more important mathematical concepts. The problem solving can, in fact, be a motivator to learn the facts with the numbers in context.

Where does one use flash cards or timed tests? When a child is developing new strategies, problem solving, strategy discussions, and games are the best tools to use. As a student becomes more confident with the strategies, particularly with the decomposition and deriving strategies, flash cards can be used to "polish off" that set of facts to develop fluency. *That fluency, however, is built upon integrated neurological pathways of derived combinations and essential algebraic ideas*.

As at the beginning of this article, place a timed test of any operation on public display. Discuss the "easy" and "hard" ones. Draw out the strategies, the mathematics. Draw connections among related ideas. Actively interrupt them, if you think they are receptive, to use more efficient strategies than the one they are using. This is the process of scaffolding, supporting them at a higher level instead of letting them operate within their comfort level. In the process, their repertoire of known facts will grow. Timed test are opportunities to pull mathematical ideas together and to explore efficiencies as well as to develop fluency. And remember, *the derived strategies used with the basic facts are exactly the same strategies needed when working with multidigit numbers.*

Summary

The Chinese see the facts between 10 and 20 as the first points where a child needs to grapple with moving above or below a ten. While exploring this initial point of the base ten system, powerful mathematical concepts and principles present themselves. Yes, we want children to eventually know as many of the basic facts as fluently as possible. Building a significant portion of that fluency upon the concepts of composition, decomposition, and underlying algebraic principles empowers students to move onto higher numbers and more sophisticated number relationships with greater ease and depth of understanding.

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Try out these ideas in your classroom

For deriving – Number Strings

Write on the board 5 + 5. Most hands will go up immediately. It pays to start with the very familiar to draw the most students into the conversation. Then write 5 + 6. Watch the reactions among your students. Who relates the information between one problem and the next? Who treats the problem as a new problem using direct modeling or a counting strategy? Draw students' attention to those who see the problems as interrelated. Make visual that 5 + 6 = 5 + 5 + 1. Write and say next, "If you know 5 + 5 and 5 + 6, what is 5 + 4?" Such number strings can be created to help nurture deriving or other strategies. Here are sample strings in multiplication and division:

Sequence 1		*Sequence 2*	
	5 x 5		25 ÷ 5
	6 x 5		50 ÷ 5
	3 x 5		30 ÷ 2
	9 x 5		60 ÷ 2
	18 x 5		60 ÷ 4

NOTE: Extending the last of the sequence into a combination just beyond the basic facts, allows students to realize what they do with smaller numbers is exactly the same as with bigger, more complex numbers.

Open Number Sentences

To draw students' attention to specific mathematical ideas, open number sentences can be effective tools.

14 + 16 = 15 + ___ *What goes on the blank line to make the number sentence true?*

This particular item will draw out the relational effect among numbers in addition as well as draw out students' understanding of equality. Multiplication examples include:

12 x 9 = (12 x 3) + (12 x ___)

or

12 x ___ = (12 x 3) + (12 x 6)

An open number sentence such as these helps students explore decomposition of number and its effect in multiplication.

Decomposition of number in subtraction

Find one of those problems most of your children consider harder to know, such as:

$$\begin{array}{r} 15 \\ \underline{-\ 8} \end{array}$$

"I know you could count back by ones to solve the problem, but is there a quicker way to solve it? Can you break the numbers apart or change the numbers in a way that would make them easier to work with?" Listen to their responses. Some will break the 8 into 5 + 3 in order to subtract the 5 to get to 10, and then subtract the remaining 3. Others will change the problem to 16 – 8 and then adjust accordingly. Either way, the goal is to encourage students to see relationships, become flexible with number and develop important mathematical ideas. "Would that strategy work with… 25 – 8?"

Games

It is well documented that children learn number combinations through games. Take a game as simple as *Chutes & Ladders.* Take the spinner away and add two dice. A child has to roll the dice, compute the total, move the playing piece that many places all the while others are watching and making sure everyone stays accurate. The games, however, are not enough if children do not see the connections between the games and other mathematical settings. It is up to the teacher to draw the children's attention to those connections. Otherwise, the child's brain will not necessarily see the two events as associated ideas.