Understanding Your Child's Mathematics

Addition & Subtraction Strategies

How You Can Help

Project for Elementary Mathematics
James Brickwedde Draft #7 2012

Your child will be asked to use a variety of strategies to solve addition and subtraction problems in the classroom. This article explains both why this is important for your child to learn and how these strategies work.

What's the difference?

When we were in school we learned one way to solve addition and one way to do subtraction using multidigit numbers. These strategies are referred to as the "American Standard Algorithms." Some students, typically on their own, developed mental math strategies that they used quietly on the side. The strategies that your child will be learning are both the standard algorithms (with some modification to better support place value) as well as the "mental math" strategies (just how they look on paper).

Why?

Place Value is one of the most important concepts your child develops in elementary school. Place value is more than just naming the value of a digit by its location in a number. It is about how a number is built and pulled apart. It is about relative relationships between numbers. It is a multiplicative relationship based on powers of ten. ("Six tens is sixty" written mathematically is 6 x 10.) Place value develops over many years. Most children do not solidify their place value understanding until fifth grade. Without solid place value concepts, multidigit multiplication and division are difficult. Decimals, fractions, percents are even harder. The "mental math" strategies tend to be stronger in the development of place value.

Algebra is now a state requirement in Minnesota for ALL students to pass by the end of eighth grade. They will no longer receive high school credit for Algebra I. This means that your child will be learning levels of math sooner than we did as children. The "mental math" strategies use key algebraic ideas while doing basic arithmetic. Developing a comfort level with these strategies allows your child to think about ideas in algebra at a 'kid-friendly' level and lay groundwork for learning in middle school or junior high.

The business world is demanding a different kind of mathematically literate worker. Rote task workers are more cheaply outsourced to foreign countries or replaced by technology. Our competition is no longer just fellow U.S. citizens, but highly skilled workers in India, China, and Europe whose *core* workers know mathematics at higher levels than the bulk of U.S. students. Business needs workers who are flexible problem solvers. We have been an innovative country. We need to stay that way. That includes allowing these 'mental math' strategies to be developed more explicitly at the elementary level.

"How Can I Help My Child?"

In the pages to follow each strategy will be explained. They may or may not be familiar to you but play around with them yourself. There are a few "rules" that are really important to keep in mind:

General "Rules" to Help Your Child

1. Watch how you talk about the numbers. The language used to describe a number is really important. Remember, *place value is the top learning priority.* So if you are adding 84 and 37, it is critical that you and your child call that "8" and that "3" (the digits being used) an 80 and a 30. Talk value when adding and subtracting numbers, not digits. It does work to go digit by digit. That's how we all learned. It works because you ignore the place value. But developing sound place value is one of your child's top goals in elementary school. Talking value raises your child's level of understanding and shortens the time it takes to solidify place value concepts. So remember, watch your language!

2. Don't place unnecessary limits on your child. This will make more sense when each strategy is demonstrated. A simple example is: if I want to add 84 and 37 together, there is no *mathematical* rule that I "*must start with the ones first."* It is just as mathematically sound to start with 80 and 30 first. I could add 84 and 30 first. Mathematically in addition it doesn't matter in which order I add the numbers and combine them. These are two algebraic ideas called the *commutative and associative properties.* Your child needs to know this idea. It opens up possibilities to add the numbers more quickly as your child gets stronger with his or her arithmetic skills. *Some* of the rules we were taught when we were in school limited us. They eventually made things harder for us in later grades. So in subtraction it is possible to take 5 from 2, but we will get to that later.

3. If you don't like the numbers you have to work with, change them! There are limits to this. The changed numbers still need to be worth the same; they need to be equal. But breaking numbers apart and using them in smaller pieces is what we do with all of our strategies all of the time. So if you want to subtract 37 from 84 I could break the 84 into 70 + 14 to make the numbers easier to work with (this is what we do in the standard subtraction algorithm) or I can break the 37 into 34 + 3, subtract 34 from 84 and then subtract 3 more. Or, I can change both numbers by adding 3 to each to get 87 and 40 and still get the same answer. (See the algebra section for why this works.)

4. Make the math visible. On some of the strategy sheets that will follow, you will notice some alternative ways of writing out the standard algorithms. For example: in subtraction, 84 - 37. When we learned it, we would cross out the "8," place a "7" above it, and then place a "1" in front of the 4 (see #1 on language). The change in the number value (see #3 on changing numbers) is hidden when we use this type of language. *While you child is first learning* these kinds of strategies, keep the math visible at all times so that the child can follow what is really happening. Write out 84 = 70 + 14 off to the side. That's important for your child to understand. Don't assume your child knows what all of our short cuts mean in terms of value. Yes, this may use more space on paper. However, once your child truly understands the strategy, the short cuts to save space will have meaning because your child will be doing more of the mathematical work in his or her head and truly understanding the changes that are being made to the original numbers.

Caution: When I say *"mental math"* strategy, this isn't to imply that your child has to *only* do this in his or her head. What follows are how these strategies, typically used by adults in *their* heads, will look like on paper. This is how your child will learn it in the classroom. You will find, however, that as your child gets better at these strategies, more and more will occur in his or her head and less and less will need to be shown on paper.

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Understanding Your Child's Mathematics

Addition Strategies, Part 1

Standard Algorithm and Tens & Ones

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The American Standard Algorithm in Addition

Also known as the 'Digit-by-Digit Strategy'

$$\begin{array}{r} {}^{1} \\ 86 \\ +\ 38 \\ \hline 124 \end{array}$$

The rules we were taught and the *classic language script* we used as we talked our way through the steps said:

- *Start with the ones place. 6 + 8 equals 14. Put down the four. Carry the one. One plus 8 plus 3 equals 12. Write the 12 in front of the 4. The answer is 124.*

This does work. However, if you remember the rules for helping your child (language of place value, limitations, breaking numbers apart, and keeping math visible) this *script*, and I emphasize *the script,* confuses many children and interferes with their place value development. There is nothing *mathematically wrong* with *the strategy.* The problem of confusion comes with the *script*! The place value was ignored. The numbers were treated as a string of single digits. If you placed the numbers in the correct location, you got the correct answer. If not, you are wrong but with no real mathematical understanding about why you were wrong.

Let's change the script and let's make the math visible and see what I mean.

$$\begin{array}{r} 86 \\ +\ 38 \\ \hline 14 \\ +110 \\ \hline 124 \end{array}$$

Your New Script:
6 + 8 is 14; Put down the 14.
80 + 30 is 110; Put down the 110.
110 + 14 is 124. The answer is 124.

The Math Made Visible:
(This is for you, not necessarily for your child)

$$\begin{array}{rcl} 86 &=& 80 + 6 \\ +\ 38 &=& 30 + 8 \\ \hline 124 &=& 110 + 14 \end{array}$$

The difference in the scripts is, in the second version, the value of the numbers and the place value is maintained *at all times*. Placing the numbers whole below the line may take up more space, but it *makes the mathematics visible* for the child while they learn the strategy. It is easier for the child to follow and learn. *Language matters. Talk value, not digits.*

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"Mental Math" Strategies: Tens & Ones

Also known as the 'Partial Sums Strategy' or 'Show All Totals'

When out in the grocery store or in a restaurant, and we need to add numbers quickly in our head, we tend to start with the larger parts of the numbers and move to the smaller part next. Children around the world, if left to their own thinking, naturally do the same thing. This makes sense as we read and say the 80 before we read and say the 4 in eighty-four.

Reminder: When I say *"mental math"* strategy, this isn't to imply that your child can only do this in his or her head. What follows is how this strategy, typically used by adults in their heads, would look like on paper as your child first learns it in the classroom.

Let's take a look at this common strategy.

$$\begin{array}{r} 86 \\ +\ 38 \\ \hline 110 \\ +\ 14 \\ \hline 124 \end{array}$$

Your Script:

80 + 30 is 110
6 + 8 is 14
110 + 14 is 124; the answer is 124.

The Math Made Visible:

(This is for you, not necessarily for your child)

$$\begin{array}{r} 86 = 80 + 6 \\ +\ 38 = 30 + 8 \\ \hline 124 = 110 + 14 \end{array}$$

Notice something! The Tens & Ones strategy is just the American Standard Algorithm in reverse. The difference is, when adults do this in their heads, place value is maintained at *all times.* The short-cut digit-by-digit language is never used.

Benefits:

1. Your child uses place value language all of the time. Children who think and track the value of the numbers are found to solidify their place value sooner and can work more mentally than children who use the digit-by-digit language/script.

2. This strategy has built in estimation. Strategically, by starting with the larger part of the number you are closer to your answer than when you start with the ones. When adding the numbers above, I know my answer should be at least above 110.

Remember, the numbers can be added and clustered in any order. These are the algebraic ideas called the *commutative and associative properties.* What is important is that your child knows that the numbers can be added in any order and you will always get the same answer.

Stages of Learning:

Your son or daughter may not know what 80 + 30 is right away. This is how he or she might sound at different stages along the way. All of these are typical of early learners:

- *80, (pause), 90, 100, 110.* They count on.

- *80 + 20 gets me to 100 plus the other 10 gets me to 110.* They break the 30 into 20 and 10 to quickly add the numbers together.

- *8 + 3 = 11 so 80 + 30 = 110.* They use what they know about smaller numbers to scale up to larger quantities. The important thing to remember is that your child *links the two relations together.* This lays groundwork for an idea in multiplication later on.

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Understanding Your Child's Mathematics

Addition Strategies, Part 2

Incremental & Compensation Strategies

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"Mental Math" Strategies: Incremental Strategy

Think of this as working 'piece-by-piece.' Some call this the 'Counting On' Strategy

This strategy has two variations. What is common to both is that a child breaks the number down into pieces then combines them one at a time. The difference is in how many pieces are created.

Variation #1: Keeping one number whole

$$\begin{array}{r} 86 \\ +\ 38 \\ \hline 116 \\ +\ \ 4 \\ \hline 120 \\ +\ \ 4 \\ \hline 124 \end{array}$$

Your Script:

86 + 30 is 116
116 + 4 (That's 4 of the 8) is 120
120 plus the other 4 of the 8 is 124

Stages of Learning: Examples of a child early use of this strategy might sound like the following:

- *86, (pause) 96, 106, 116, (pause), 117, 118, 119... 124 (Counts up by ones eight times.), or*
- *86, (pause) 96, 106, 116, (pause), plus 4 gets me to 120, plus another 4 gets me to 124.*

Variation #2: One piece at a time

$$\begin{array}{r} 86 \\ +\ 38 \\ \hline 110 \\ +\ \ 8 \\ \hline 118 \\ +\ \ 2 \\ \hline 120 \\ +\ \ 4 \\ \hline 124 \end{array}$$

Script difference:

Same as above. It is just that the numbers are broken down into place value parts first and then added piece by piece. *With time and practice, most of these small steps are quickly done mentally and speedily.*

Benefits:

1. Your child learns how to add a ten on to any number (86 + 10), not just from a round decade number like 80 + 10.
2. Your child learns the fact families of the single digit numbers (example: 8 = 4+4 = 5+3 = 6+2...)
3. They break these single digit numbers apart in order to quickly get to the next ten thus building fluency with organizing around ten.

"Mental Math" Strategies: Compensation Strategies

Again, there are two variations. This time, however, the variations use two different ideas based on the concept of equality. The first strategy uses rounding to make the problem unequal to the original problem. The second version makes changes to both numbers to make sure everything stays equal at all times.

Version #1:

$$\begin{array}{r} 86 \\ +\ 38 \\ \hline \end{array} \quad \textit{round off to:} \quad \begin{array}{r} 86 \\ +\ 40 \\ \hline 126 \\ -\quad 2 \\ \hline 124 \end{array}$$

Your Script:

I add 2 to 38 to make it 40 which is an easier number to work with.
86 + 40 = 126
Since I added 2 to 38, I have 2 too many in my answer so I need to subtract 2
126 - 2 = 124 My answer is 124.

Version #2

$$\begin{array}{r} 86 \\ +\ 38 \\ \hline \end{array} \quad \begin{array}{l} \textit{take 2 off of } 86 \\ \textit{and add it to 38} \end{array} \quad \begin{array}{r} 84 \\ +\ 40 \\ \hline 124 \end{array}$$

Your Script:

To make both numbers easier to work with I take 2 off of the 86 and add it to the 38. This will give me the same answer as the original problem.
84 + 40 = 124

Important Mathematical Ideas:

To do either of these strategies well, a child needs to have a growing idea about what the 'equals' (=) sign means. Most children think = means the 'answer comes next.' This is not correct and severely limits his or her ability to work with important algebraic ideas later on. Variation #2 is built on the idea that I can keep the values equal even if I recombine the numbers in an equivalent amount. Example:

$86 + 38 = (84 + 2) + 38 = 84 + (2 + 38) = 84 + 40$

Version #1 requires a slightly different thinking about the issue of equality. The change made by rounding off just one number makes the problem become unequal ($86 + 38 \neq 86 + 40$). The changed numbers, however, are a lot easier to work with. What a child needs to focus on is how does he or she undo the change. Since the one number was increased in size, the effect is the initial answer is too big so it needs to be made smaller by the amount originally added: $86 + 38 = 86 + (38 + 2) - 2 = (86 + 40) - 2$

There is a lot of algebraic ideas at work in both of these versions; ideas about equality being the most critical. But there is another skill being developed called 'relational thinking.' This skill allows a child to think about the effect of changes on a number and how it affects the outcome. Is it still of equal value? If I move this number to the other side of the equal sign to make the problem easier, what do I need to do to keep everything equal? As I said, we are getting into some algebra and elementary students are capable to grasping these ideas.

Limitations and Cautions:

- Don't limit your child by focusing on a 'rule' such as, *"If you add a number you subtract."* While true in addition, your child will run into problems when he or she gets to subtraction. Focus on the *effects* of the changes to the numbers, not the surface rule.
- The idea of this strategy is easy and makes sense to most children. Doing it initially is hard because how to readjust the numbers requires careful thinking. Use pictures or models that will help make the mathematics visible.

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Understanding Your Child's Mathematics

Subtraction Strategies, Part 1

Standard Algorithm and Tens & Ones

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We outlined three basic strategies in addition - Tens & Ones/Standard American Algorithm, Incremental, and Compensation. Each of these has a counterpart in subtraction with one extra strategy; that of adding up to find the difference between the two numbers. First, let's take a look at the strategy we were all taught in school and understand how it really works.

American Standard Algorithm in Subtraction

As with the addition standard algorithm, the struggle children have around learning it is not there is anything mathematical wrong with the way the algorithm works, the script we use to describe the steps of the procedure hides the mathematics. Let's review the procedure as we were taught it. We will use the same numbers as before, but this time subtract.

$$\begin{array}{r} {}^{7}\\ 8\,{}^{1}6 \\ -\;3\,8 \\ \hline 4\,8 \end{array}$$

Classic Verbal Script:

- *I can't take 8 from 6 so I borrow 1 from the 8, make it a 7, put the 1 in front of the 6 .*
- *16 take away 8 is 8. Put down the 8.*
- *7 take away 3 is 4. Put down the 4. The answer is 48.*

This should sound familiar. At its core, *the script that is used,* and again I emphasize *the script,* asks you to ignore the *place value,* act upon the numbers *as a string of single digits*, and, if you place those digits in the correct location, you get the correct answer. Many children have been interviewed who honestly believe that all they have literally taken from the "eight" is a one, and the only reason the six becomes "sixteen" is because you placed the "one" in front of it. They do not understand mathematically how the 86 is altered. They do not recognize that the "^{7} and ^{1}6" is in reality 70 + 16 and that it is still worth 86. Remember the "rules" for helping your child? Let's look at the same algorithm but with *the place value language maintained, how the numbers were broken apart to make them easier, and the mathematics made visible.*

$$\begin{array}{r} 86 \\ -\,38 \\ \hline 48 \end{array} \quad \begin{array}{l} = 70 + 16 \\ = 30 + \;\;8 \\ \hline = 40 + \;\;8 \end{array}$$

New Script:

- I don't have enough to take 8 from 6 so *I am going to break apart 86 into 70 and 16 to make 86 easier to work with. I also break 38 into 30 and 8.*
- *16 take away 8 is 8. 70 take away 30 is 40. 40 plus 8 is 48. The answer is 48.*

This script describes why and how 86 is broken apart. Breaking apart numbers is a very important idea and skill for children to grasp. (See Rule 3: If you don't like the numbers...) Expanding the math off to the side helps make the numerical changes more visible. I don't expect children to go into adulthood having to do this expanded version, but I do anticipate them having to stay in this stage for a while to solidify their place value. They can eventually learn the more collapsed version with its shortcut notations. The difference, however, is that his or her mental script will reflect the legitimate changes to the numbers and maintain place value at all times.

"Mental Math" Strategies: Tens & Ones Strategy

Also known as 'Partial Differences'

I have paired the 'Tens & Ones Strategy' with the 'American Standard Algorithm in Subtraction' because both use similar place value concepts. The Standard Algorithm makes changes to the "top" number in order to then subtract the *ones* and then subtract *the tens.* Let's see what happens if a child tries and starts with the tens first as can easily be done in addition.

$$\begin{array}{r} 86 \\ -\ 38 \\ \hline 50 \end{array}$$

Script:
80 minus 30 is 50.

So far, so good! But now, there is a *potential problem.* I say potential because it's only a problem if you are limited to thinking "you can't take 8 from 6." (See Rule 2: Don't Place Unnecessary Limits on Your Child.) There are two ways of thinking about the situation of 6 - 8: **one** is, if you know that *zero can be bridged,* meaning there are numbers on the other side of zero, your child can use negative numbers. **The second is asking the question,** "*what am I short?*" Let's continue with each scenario one at a time.

Scenario 1: *"Going Negative"*

$$\begin{array}{r} 86 \\ -\ 38 \\ \hline 50 \\ -\ \ 2 \\ \hline 48 \end{array}$$

Script:
- *80 minus 30 is 50.*
- *6 minus 8 is negative 2 ($^-2$)*
- *50 and $^-2$ is 48. The answer is 48.*

Note that technically your child is adding 50 + $^-$*2* but that has the same effect as 50 - 2. This is a long term association involving *inverse elements,* but I would let that sit in the background for now and let your young child use the equivalent positive expression.

Scenario 2: *"What am I short?"*

$$\begin{array}{r} 86 \\ -\ 38 \\ \hline 50 \\ -\ \ 2 \\ \hline 48 \end{array}$$

Script:
- *80 minus 30 is 50.*
- *6 minus 8, I can take away 6 but I'm short 2, so the 2 has to come out of the 50. 50 minus 2 is 48. The answer is 48.*

So, if your child is not comfortable thinking about negative numbers, your child is usually very aware what he or she is short of if they have six and want eight.

Benefits:

Using the Tens & Ones Strategy in subtraction maintains place values at all times. Using it, however, does compel your child to think about other mathematics different than the Standard Algorithm. Your child needs to either think about how zero can be crossed over into the realm of negative numbers, or your child needs to understand the difference between two numbers, do you have enough or are you short an amount (in debt so to speak). With this last idea, if I am short an amount, I need to take it out of what I have left. In this case it has to come out of the remaining fifty. Saying 6 - 8 can't be done sets up huge misconceptions for later work with algebra.

The strict use of the Standard Algorithm typically would say that 50 - 2 cannot be done because you can't take 2 from zero. But that is not efficient! I want all children to look at 50 as a whole number and be able to know what two less would be. That is a reasonable expectation!

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Understanding Your Child's Mathematics

Subtraction Strategies, Part 2

Incremental and Compensation Strategies

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"Mental Math" Strategies: Incremental

Also described as working 'Piece-by-Piece"

This strategy has some advantages in subtraction over the Tens & Ones approach. Finding the subtraction of six minus eight (6 - 8) as too confusing from a Tens & Ones perspective, some children begin to work with each "piece" of the number one at a time. The big deal in subtraction is how do you go from one decade to the decade below. Example: In 24 - 6, the mathematical idea is how do you get from the twenties to the teens without necessarily breaking the 24 into 10 + 14 as you do in the American Standard Algorithm. You have some choices. Here are two typical ways children negotiate this.

Version #1a & 1b:

a)	b)
86	86
- 38	- 38
50	50
- 8	+ 6
42	56
+ 6	- 6
48	50
	- 2
	48

Script:

- *80 minus 30 is 50.*
- Now you have two choices, deal with the 8 first or deal with the 6 first.

a) The 8 first: *50 - 8 is 42. I have add the six back on so 42 + 6 is 48. The answer is 48.*
b) Now the 6 first: *50+6 is 56. 8 = 6+2 so 56 - 6 is 50, minus 2 is 48. The answer is 48.*

What's in Common? A child who works this way is looking at 86 and 38 by their place value components. The child also is exploring the idea that the order of the operations can be altered. There is no limitation here in that regard. But instead of breaking 86 into 70 + 16 as in the standard algorithm, the child is breaking the 38 into 30 + 6 + 2 (version 1b). The adage of, "If you don't like the numbers you have, change them!" is being used here just as in the Standard Algorithm, it's just that the numbers be altered is the bottom number not the top number. Both choices work!

Version #2:

86
- 38
56
- 6
50
- 2
48

Script:

- *86 minus 30 is 56.*
- *8 = 6+2 so I will subtract it in two parts, 56 - 6 is 50 (that's 6 of the 8), minus the 2 (the last of the 8) is 48. The answer is 48.*

Here the child keeps the larger number whole and took of the tens and then the ones of the bottom number. This saves time compared to the first versions, but the mathematical ideas are the same.

Early Versions: When children first try these strategies, they may need to count back one ten at a time (*86, 76, 66, 56, (pause) 55, 54, 53...48).* This is normal! As their number sense matures, and they are pushed to think about saving themselves time, they come to work in bigger and bigger chunks of numbers and become very fast.

"Mental Math" Strategies: Compensation

This is the strategy where rounding numbers to make them easier and quicker to work with becomes the skill to be cultivated. Like in addition, your child needs to keep track of how the relationships between the numbers are changed if just one of the numbers is changed or if both numbers are changed. You really cannot get by with simple rules here. Subtraction distorts the changes differently than in addition. Let's see how this works with the numbers we have been using.

Version #1:

$$\begin{array}{r} 86 \\ -\ 38 \\ \hline \end{array} \quad \textit{round off to:} \quad \begin{array}{r} 86 \\ -\ 40 \\ \hline 46 \\ +\ \ 2 \\ \hline 48 \end{array}$$

Script:
- *I round off 38 to 40 to make the problem easier to work with.*
- *86 minus 40 is 46.*
- *But with 40 I took 2 too many so I need to add 2 back on so 46 plus 2 is 48. The answer is 48.*

Notice how this is different than in addition. By making the number larger, more was taken away. If you look at this on a number line, by adding 2 you *shorten the difference between* the two numbers, so you need to add the two back on to lengthen the distance to the original.

Version #2

$$\begin{array}{r} 86 \\ -\ 38 \\ \hline \end{array} \quad \begin{array}{l} \textit{add 2 to } 86 \\ \textit{add 2 to 38} \end{array} \quad \begin{array}{r} 88 \\ -\ 40 \\ \hline 48 \end{array}$$

Script:
- *If I add 2 to both numbers I get 88 - 40 which will have the same difference as 86 - 38. 88 minus 40 is 48. The answer is 48.*

Notice that this compensation strategy is solely based upon the *difference between* two numbers rather than the *take away* model with which we are more accustomed. 86 - 38 = 88 - 40. Look at this on a number line to see why this works all the time. Important ideas of equivalency are being developed here. This strategy certainly takes advantage of the rule, "If you don't like the numbers you have to work with, change them!"

There is another version, less used, but worth noting. Paying attention to it re-enforces the caution about not focusing on memory rules but instead focusing on how the relationship between the numbers are affected. The two above versions were based upon changing either just the lower or both numbers. What would happen if we were to just change the top number?

Version #3:

$$\begin{array}{r} 86 \\ -\ 38 \\ \hline \end{array} \quad \textit{round off to:} \quad \begin{array}{r} 90 \\ -\ 38 \\ \hline 60 \\ -\ \ 8 \\ \hline 52 \\ -\ \ 4 \\ \hline 48 \end{array}$$

Your Script:
- *I add 4 to 86 to make it easier to work with. 90 minus 30 is 60. 60 minus 8 is 52. But by making 86 ninety, I have 4 too many so I need to take that 4 off so 52 minus 2 is 50 minus another 2 is 48.* (Notice the mix of strategy ideas from other those described on other pages)

Notice here the similarity with the addition compensation decisions. The adjustments are different than when the bottom number is rounded off. Simple memory rules such as "if I add it I have to subtract it," can trap you into making an error. It's better to track the changes in the relations than rely on simple surface rules.

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Understanding Your Child's Mathematics

Subtraction Strategies, Part 3

Adding Up: The Inverse Operation

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Adding Up: "The Difference Between"

(Or think of this as the "Giving Back Change" Strategy)

Consider the following problem, *"TJ earned $400 over the summer mowing lawns for families in the neighborhood. At the end of the summer, he bought a bicycle for $294. How much money does TJ leftover?"* Now, the context of this problem is a subtraction, "take away" structure. But let me write it out in a column format.

```
  $400
-  294
```

Yes, I could do the American Standard Algorithm for Subtraction by `changing the 400 into 300 + 90 + 10 and then subtract each part of the 294 away from each of the top number parts. I could use the Incremental or Tens & Ones strategies. But given these numbers many individuals will naturally add up. They change the problem into $294 + ____ = $400. The script would sound like the following: *"294 + 6 gets me to 300. 300 plus another 100 gets me to 400. The answer is $106."*

Two important mathematical ideas are critical here. The first is that *any* subtraction problem can be restructured into an addition problem. This is a fairly sophisticated abstract idea. Where most children see adults use this strategy is in making change. The "School" version of making change is to use subtraction, but that is not that case in the world of commerce. You add up to make change. Once children realize this, doors are open to think more flexibly about numbers.

"Take Away" and the "Difference Between"

The second mathematical idea that is important for your child to develop is that there are two ways to think about a subtraction problem, one is the very familiar "Take Away" strategy; the other is the less familiar "Difference Between" strategy. Consider this problem, *""The score of the basketball game was 63 to 47. By how much did the winning team beat the losing team?"* Most of us were trained in school to think of this as a "Take Away" problem - "63 take away 47 is..." - and write it out as 63 - 47. But if I think of this as the difference between two numbers like the comparison context of the problem implies, then I could solve it as 63 - ___ = 47, or I could turn the problem into an addition problem and solve it as 47 + ___ = 63. So, let's practice these three strategies. Each of the two subtraction approaches has been described earlier.

American Standard Algorithm
(made visible)

```
  63 = 50 + 13
- 47 = 40 +  7
  16 = 10 +  6
```

Difference Between Strategy
(Counting back: Incremental)

63 - <u>10</u> → 53 - <u>3</u> → 50 - <u>3</u> → 47; 10 + 3 + 3 = 16

Difference Between Strategy
(Adding Up: Incremental)

47 + <u>3</u> → 50 + <u>13</u> → 63
13 + 3 = 16

Which one is better?

As they say around here, *"It all depends on if you are from Minneapolis or St. Paul!"* meaning you are both looking at the Mississippi River but each from your own side. Some of this is preferred choice. However, sometimes there is a legitimate case for one being more efficient than the other. In the case of $400 - 296, I would argue that either of the Difference Between Strategies are quicker and more efficient. *(We will talk about efficiency soon.)* The numbers 63 - 47 are so close together that looking at these numbers in terms of their difference is arguably more efficient. **The important mathematical idea, however,** is that your child needs to understand **both** the Take Away **and** the Difference Between strategies. To make an efficient choice, you need to be fluent with all three strategies.

But what about really big numbers? Does Adding Up work there?

Let's practice using some classically very unfriendly number combinations. I present the problem as a subtraction problem as it would potentially appear on a test or worksheet. The addition components are written off to the side as you might jot them down to keep track of your thinking.

5,362	2,362
- 2,794	206
2,568	

Script:

• (Draw a line between the two numbers. The line represents, in this case, the nearest thousand to 2,794.) *Say:* "Plus 6 gets me to 2,800, plus 200 gets me to 3,000. So that's 206. (Write it down under the line.) From 3,000 to 5,362 is 2,362. 2,362 + 206 = 2,568. The answer is 2,568.

If you choose to use the Adding Up: Difference Between Strategy, it can be very fast and very efficient. Notice, however, in the script that to become good at it, you have to be good at moving in chunks of numbers using place value skills and getting to landmark numbers. These skills need to be consciously built.

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Understanding Your Child's Mathematics
Building Number Sense Skills & Fluency

Project for Elementary Mathematics
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"Get to a Number"

Remember at the beginning of these conversations about how **place value** is so foundational to everything your son or daughter needs to build his or her number sense on? If your child, as well as yourself, have begun to consistently call numbers by its value rather than just by its digit name, building this skill of "getting to a number" in ever more efficient chunks is easier. If all numbers are strings of single digits, this is hard. Children who struggle to do this next series of activities usually have weaker place value knowledge.

Tools: • A deck of cards with the picture cards set aside; • A cut up "hundreds chart" (This gives you a deck of cards with the numbers 1-100)

Working Up

Get to 10 (Or the next 10): Using a deck of cards, turn over a card and ask, *"How much to get to 10?"* Extension: Do the same thing and ask, *"How much to get to 20?"* Now take the cut up 100 chart cards, turn one card over and ask, *"How much to get to the next ten?* Extension: Do the same thing, say 46 is drawn, and ask, *"How much to get to 60?"* meaning two tens above the number.

Get to 100, The Next 100, The Next 1000: Using the cut up hundreds chart deck of numbers, flip one card over at a time and ask, *"How much to get to 100?"* **Early learners** will need a way to visually see the numbers. Use the following arrow notation to help: Example: 67, How much to get to 100? 67 + 3 → 70 + 30 → 100, the answer is 33. Early attempts by children typically require going up only one ten at a time (70, 80, 90, 100). As a child's number sense grows, the ability to work in larger and larger chunks gets easier. Some numbers need to be jotted down on paper, other parts are done in the head.

Getting from A to B: Draw two number cards from the 100-deck (say 23 and 56) and ask, *"How much to get from 23 to 56?"* With arrow notation, and at its most mature stage, might look like this: 23 + 7 → 30 + 26 → 56, 26 + 4 + 3 = 33 The answer is 33. A child could also do the following: 23 + 30 → 53 + 3 → 56, The answer is 33. This is one of those Minneapolis, St. Paul things. Both work. It's a matter of preference and judgment. Getting fluent at this stage turns any subtraction problem into a quick Adding Up problem.

Working Backwards

Go Back from a Decade: Pull out the decade cards from the 100-card deck (10, 20, 30...) and place them in a deck of their own. Draw a card from the deck of regular playing cards. Say, *"Go back..."* Example: Draw 70. Draw 4, say, *"Go back 4."* This is an alternative reinforcement of facts to 10 but from a subtractive point of view.

Go Back from a Number: Draw a card from the 100-card deck and say, *"Go back..."* Example: Draw 82. Draw 6. Say, *"Go back 6."* The goal is to have the child think about how to break the 6 apart quickly, particularly by

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getting to the nearest ten and then, using their ten facts, go back into the decade below. *"Go back 2, back 4."* 82 - 2 → 80 - 4 → 76.

Go Back from 100: Draw a card from the 100-card deck and say, *Go back...* Example: Draw 47. Say, *Go back 47 from 100.* 100 - 40 → 60 - 7 → 53. It is acceptable for the student to change this into an addition problem. However, it is worthwhile to develop the skill backwards in chunks.

Get From B to A: Draw two cards from the 100-card deck and say, *"Go back from... to..."* Example: Draw 58 and 35. Say, "*Get from 58 to 35."* There are two subtractive approaches to this game. The Difference Between strategy, as the game is set up to be, would look as follows: 58 - $\underline{8}$ → 50 - $\underline{15}$ → 35; 15 + 8 = 23; the answer is 23. A Take Away strategy can also be used: 58 - 30 → 28 - 5 → 23; the answer is 23. While the game is set up to work backwards from one number to the next, if a child quickly turns the numbers around and add up, let them. It is a mathematically sound strategy and represents a flexibility that should not be discouraged: 35 + $\underline{5}$ → 40 + $\underline{18}$ → 58; 18 + 5 = 23; The answer is 23.

Arrow Notation & The Equal Sign

Notice that I have been using arrows in the notation system to represent steps taken to solve a problem. Many children, and even some adults, like to show their actions in a continuous flow. Let's take one of the examples above: *'Get from 35 to 58.'* Some might write their steps this way:

$$35 + \underline{5} = 40 + \underline{18} = 58;\ 18 + 5 = 23;\ \text{the answer is } 23$$

This is not a mathematical correct use of the equal sign. The equal sign is a relational symbol. All the values on one side of the equal sign need to be worth the same as the values on the other side. So while 35 = 5 does equal 40, 35 + 5 *does not equal* 40 + 18. It is important for even young children to make this distinction.

Equality chains can and do exist. Example: 18 + 5 = 18 + 2 + 3 = 20 + 3 = 23. It is perfectly legitimate to use the equal sing in this manner. Notice, however, that each set of values, across all combinations across the chain, combines to make the same value. 'Equals' *does not mean 'the answer!'* "Equals means the values are worth the same.

The arrow notation allows your child to use an 'open number line' image to help organize his or her thinking. The values added or subtracted, can be underlined as above to help distinguish which values one needs to combine afterwards. But those values can also appear *above the arrow* to make them more distinguishable. Example:

$$35 \xrightarrow{+5} 40 \xrightarrow{+10} 50 \xrightarrow{+8} 58;\ 5 + 10 + 8 = 23$$

The arrows can also be used in reverse fashion to help accentuate movement towards zero on the number line.

$$35 \xleftarrow{-5} 40 \xleftarrow{-10} 50 \xleftarrow{-8} 58$$

The arrow notation is merely a way to capture ones actions without compromising the use of the "=" sign. That's all. It holds no more significance than that.

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Understanding Your Child's Mathematics

Addition & Subtraction Summary

Why Bother? What's the Advantage?

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Let's Review:

The Four Rules to Help Your Child:

1. Watch how you talk about the numbers.
2. Don't place unnecessary limits on your child.
3. If you don't like the numbers you have to work with, change them!
4. Make the math visible.

The Mathematics

Place Value: Without it, numbers are just a pile of ones! And as numbers get bigger, counting everything by ones gets a little tedious. Place value matures over time, in fact, deep elements of it do not fully mature until a child is in fifth grade. Place value is more than just saying that the "three" in 4,327 is worth 300. But that "three" is also 300 ones, 30 x 10, 3 x 100, as well as multiple other sub-parts such as 200 + 100, or 100 + 100 + 100. Place value is about multiplicative relationships. How a number is built (20 + 3, or 20 + __ = 27), how a number is broken apart (17 – 7), and how a number is compared (30 compared to 34, 23 compared to 33), are among the different ways numbers need to be thought about where elements of place value are used. Without a strong, deep sense of place value, your child is sunk! This is one of the reasons why it is important to describe *the value of the numbers* when adding or subtracting rather than calling them *by their digit names.* Keep the place value the focus. Shortcuts that ignore place value do more long range damage than short-term gain.

Breaking numbers apart: That 10 is made up of 7 + 3 or 6 + 4 or any of its other sub-parts is important for a child to understand. If it's not easy taking 18 away from 45, I need to know that I can break down the 45 into something easier (typically 30 + 15) or I can break down the 18 (typically 10 + 5 + 3). So much of what happens in mathematics is knowing different ways numbers can be broken down into smaller, easier to work with combinations.

Multiple Strategies: Each of the different addition and subtraction strategies draws on some different mathematical skills. Taking 84 - 30 (subtracting a group of ten from any number) is a slightly different skill than taking the 30 from the 80 and then adding the 4 back on. The Tens & Ones (84 - 36, 80 - 30 = 50, 4 - 6 = ⁻2) strategy in subtraction causes an individual to either think about negative numbers (and that there is life on the other side of zero!) or think about how I am "short 2" and that that 2 needs to be taken out of the remaining 50. By being fluent in several strategies, a child learns a broader range of number skills. He or she thinks about numbers from many different angles instead of just those ideas around one single strategy. The child becomes more flexible working with the numbers and can make many more judgments about how to work the numbers. By not putting artificial limitations on children ("Such as, *"You can't take 6 from 4."*) doors are left open to be explored when the time is ripe.

The Math, The Algebra: When you go to add 54 and 38, it does not mathematically matter whether you add the ones first or the tens first or only one group of ten to the other whole number. First, you are breaking the numbers into place value components. Second, you are using the commutative property of addition to move the numbers around. So in this one instance, the place value was maintained, there were no limits placed on the order of the addition and the math was kept visible. An algebraic property, the commutative property, was used. Having your child explore the commutative property is important for two reasons: first, it can make chunking combinations of numbers together easier, and, two, it has its limits. 2 + 18 may equal 18 + 2 but 18 - 2 does not equal 2 - 18. There is a *mathematical* limit that *is necessary* to explore and comprehend. The order of operations may *sometimes* (there are definite limits here) be tinkered with. The Incremental Subtraction Strategy is one such place where this can happen. Recall, 86 - 38. I can break this down into 80 - 30 + 6 - 8, but just as easily I could do 80 - 30 - 8 + 6. Or, I could even, if it was really worth it, do 80 - 8 - 30 + 6. I will get the same answer each time. But be careful, it does not work with all mixes of operations and that is why parentheses ( ) were invented!

When we looked at the compensation addition strategies where we did some rounding of numbers to make the addition easier, issues of equality were important. The number 86 + 49 may be easier to add if you think of it at first as 86 + 50, but 86 + 49 does not equal 86 + 50. I need to subtract the one number I added on to bring the numbers back into equilibrium. On the other hand, if I am good at breaking numbers apart into equal sub-parts, I can redistribute the values of the numbers to make them easier to work with. 86 + 49 can be broken down into 85 + 1 + 49. The 1 can then be added onto the 49 to make it 50. 86 + 49 does equal 85 + 50. To do that I need to understand about decomposing numbers, the associative property in addition (I can added neighboring numbers in any order), and I maintained equivalent relationships. All of this uses elements of algebra.

We will talk more about the algebra underlying the arithmetic one does see in a later section.

The Advantage:

Your son or daughter will be a very different everyday mathematician than you or I if they learn and focus on the variety of strategies described so far. Your child's place value will mature earlier and more soundly. His or her ability to make judgments about numbers and be flexible will be stronger. If the underlying ideas of the mathematics are made visible, your child's algebraic thinking will be more visible. You will hear the difference as they work and talk about the numbers. They will be ready for the workplace as they get older because they will be able to do and understand more. And in our economy, that's an advantage!

But what about multiplication and division? What's expected there? Let's take a look.

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Understanding Your Child's Mathematics

Multiplication Strategies: Part 1

Early Strategies

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Young children initially approach multiplication through repeated addition. Researchers have shown that kindergarteners can solve a problem such as, *3 cookies on a plate; there are 4 plates, how many cookies are there on all of the plates?* Children typically use blocks or pictures to organize what's there and count all of the items singly to find the answer of 12. They might start skip counting by threes or adding successive threes: "*3 + 3 → 6 + 3 → 9 + 3 → 12.*" More accomplished children might start making doubles and then doubling the doubles: *3 + 3 = 6, 3 + 3 = 6, 6 + 6 = 12.* These are all examples of early stages of solution strategies for multiplication problems. Typically at this point, however, most of the children are primarily thinking additively.

So what is multiplication then if not merely "fast adding"? How can you tell if your son or daughter is beginning to think multiplicatively instead of just additively? Let's use money as an example. Those of you who have spent time teaching your child to comprehend and count money know from experience the pitfalls a young learner goes through. A dime, of course, is worth the same as ten pennies. However, a pile of 10 pennies *looks* a whole lot more then 1 dime. There are certainly more objects with 10 pennies. Those 10 pennies are heavier than that 1 dime. Never mind that the nickel is bigger than the dime but worth half as much. Multiplication is about making a unit out of a collection of subunits. The new unit becomes the countable object. But to truly be thinking multiplicatively a child needs to mentally keep track of both the count of the number of tens and the number of ten ones simultaneously. In the context of the dime, your child needs to securely remember that those 4 dimes are also 40 pennies and that they are all *worth the same.*

Let's look at skip counting by threes to help visualize a child's need to keep count of two simultaneous counts. Counting by threes takes a little longer for children mostly because fewer things in our lives come naturally in threes as oppose to the more common counts of twos, fives, and tens. Let's use the scenario of, *Six bags of doughnuts with three doughnuts in every bag. How many doughnuts are in all of the bags?* A child in the earlier stages of skip counting will count up as high as they can remember the rote sequence and then revert to counting one by ones in a series of bursts of three. While counting, however, they typically raise one finger for every count of *one group of three.*

Skip Counting

Example:	*Sequence of three ones:*	*3,*	*6,*	*9,*	*12,*	*13,14,15,*	*16,17,18*
	Finger count of each group of three:	*1,*	*2,*	*3,*	*4,*	*5,*	*6*

If words were placed to all of the mental tracking, and some early skip counters actually will talk out loud this way until the tracking becomes automatic, *"One bag it three, two bags is six, three bags is nine…"* until they are up to 18 doughnuts. The answer then becomes its own interesting puzzle for very early learners. Is there answer 18, meaning the number he or she last said, or is it 6, the number of fingers held up?

Doubling
Complex doubling

Part two – distributive property

Factoring and associative property

Multiplication & Place value (counting zero pattern)

Learning Basic Fact Strategies

Building capacity (place after the division section)

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Understanding Your Child's Mathematics
Division Strategies
American Standard Algorithm in Division &Partial Quotients

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The American Standard Algorithm in Division is the method that the majority of us learned when we were in school. With all of the previous standard algorithms we have discussed in addition, subtraction, and multiplication, the argument has been made that the mathematics underlying each of those algorithms is sound but *the script* we use to talk our way through it inhibits place value development and masks the mathematics. Fix the script and the algorithms become more mathematically powerful. The American Standard Algorithm in Division has problems once place value is emphasized. The script we use is not merely masking the place value it is based on a false premise in its very initial step. Let me explain. Here's how we learned division:

$$\begin{array}{r} 31\ R\,6 \\ 8\,\overline{)254} \\ \underline{-24} \\ 14 \\ \underline{-\ \ 8} \\ 6 \end{array}$$

The Script:

- *8 does not go into 2, move over a space*
- *8 goes into 25 three times. Place a three above the 5*
- *3 times 8 is 24*
- *25 minus 24 is 1. Bring down the 4*
- *8 goes into 14 one time. Place a 1 above the 4*
- *14 minus 8 is 6,*
- *The answer is 31 remainder 6*

Sound familiar? (This is often called the "Gozinta" Strategy as in "goes into.")

Now let's look at this from the angle of maintaining place value knowledge at all times. Let's review that first step:

- *"8 does not go into 2"*

Let's put the place value back into the language and focus on the mathematics of the number involved:

- *"8 does not go into 200"*

Well, that is not a true statement. In fact 8 does go into 200 25 times! Once the focus is on the place value in the number, the thinking behind the standard algorithm falters. It changes into a significantly different conversation. One can argue that once you get to the second step, "8 goes into 25 3 times," that if you say "8 goes into 25 tens" the place value can be used. However, the very first declaration of ignoring the 200 by pretending it is a two is a serious mathematical flaw. It is important for children to know that 8 can go into 200 a significant number of times.

Let's look at an alternative called the "Partial Quotient" strategy. The difference with this strategy is it allows room for estimation and building closer to the answer instead of needing to be highly precise immediately. The precision is fine tuned over time. Its other benefit is that is maintains the true value of the numbers involved.

Partial Quotients Strategy

The Partial Quotient strategy starts with an estimate of the largest possible amount a child thinks he or she can start with. The more sophisticated the child's number sense, the more precise they can become. The first presentation of this problem will talk through how a more advanced child might sound. I will then walk through more emergent scripts later that demonstrate earlier stages of learning.

```
8 )254 |  20
  -160 |
    94 | + 11
  - 88 |  31 R 6
     6 |
```

The Script:

- *10 8s would be 80 so 20 8s is 160 so that's closer*
- *254 minus 160 is 94*
- *11 8s is 88 which is the closest I can get*
- *94 minus 88 is 6, and no more groups of 8 are possible*
- *So the 20 8s plus 11 8s is 31 8s and there is a remainder of 6*
- *The answer is 31 whole groups of 8 with 6 out of another group of 8 leftover*

This is a very different script. Notice how the place value and number sense is always maintained. Notice in the script how the opportunity to scale up what you know about smaller amounts (8 x 10 is 80) to determine if a larger quantity can be used (8 x 20 is 160). The example above could become even more efficient by recognizing that 30 tens is 240 thus saving even more steps. ***The flexibility of this strategy is that it grows with your child.*** If you are helping your child at an earlier point in the learning, this strategy allows them to calculate in smaller intervals and still determine the correct answer. Let's look at a potential earlier version of this same problem.

```
8 )254 | 10
  - 80 |
   174 | 10
  - 80 |
    94 | 10
  - 80 |
    14 | + 1
  -  8 | 31 R 6
     6 |
```

The Script:

- *10 8s would be 80*
- *254 minus 80 is 174*
- *I can do another 10 8s*
- *174 minus 80 leaves 94 left*
- *I can do another10 8s*
- *94 minus 80 is 14*
- *Only one 8 is in 14*
- *14 minus 8 is 6, no more groups of 8 can be formed*
- *10 + 10 + 10 + 1 is 31 8s with a remainder of 6*
- *The answer is 31 whole groups of 8 with 6 out of another group of 8 leftover.*

The Benefits:

- Place value is maintained and nurtured at all times
- Partial Quotients allows the child to increase his or her number sense over time without having to be exact from the start.
- The child can grow into more efficient number sense
- There are no misleading mathematical statements that will hinder future learning
- More sophisticated multiplicative thinking is nurtured in this strategy than in the American Standard Algorithm for division

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Understanding Your Child's Mathematics

Division Strategies

Partial Quotients with Large Numbers & Decimals

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Let's now look at how partial quotients works when dividing larger numbers. Remember, watch your language; place value matters. Remember, you don't need to be exact at each step. You can be reasonably close and then narrow the precision as you proceed. The stronger your child's number sense is and his or her ability to "scale up" the fewer steps it will take. Having your child ask him or herself if the first estimate is close, or can they double or triple that estimate to get even closer, are the types of reflective questions that increases the precision over time.

Below is the same problem done at different levels of precision. Look at the types of questions asked that potentially increases the level of efficiency.

28)3,254	100
-2,800	
454	15
- 420	
34	+ 1
- 28	116 R 6
6	

The Script:

- *28 hundreds is two thousand eight hundred*
- *With 454 left, 10 twenty-eights is 280. Double that (560) is too large but half as much would be 140 + 280 so 15 twenty-eights 420*
- *With 34 left, that's one more 28, leaving 6 remaining so the total is 116 whole groups of 28 with 6 leftover*

Notice the use of 100 as a landmark. Notice how the language what 28 hundreds are leads directly to the answer one is looking for. Notice the scaling up around 10 groups of 28, doubling would be too big so half as much (5) is found to combine to make 15 groups of 28. That ability to scale up is the skill that you are looking to nurture over time with your child.

A more emergent version of this might still be able to start with 100 but an even more emergent strategy might only be able to double what 10 groups of 28 are. Let's look at such an emergent attempt.

28)3,254	20
- 560	
2,694	20
- 560	
2,134	20
.	
.	
.	

Notice the steps followed are the same but with a weaker number sense it merely takes longer. With more steps, there is an increase in the possibility to make an arithmetical error. In a problem like this, it is opportune to actually interrupt the child at, say, the third repetition and ask them, "Are you close or still really far away? Could you double or triple the 20 and save yourself some time?"

That kind of prompt nurtures the scaling up thinking necessary to become more and more efficient in the steps taken to become proficient with this strategy. They can grow into the strategy.

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Partial Quotients Strategy with Decimals

Division with decimals is typically a topic at fifth grade. The first place students need to grapple with this idea is in solving the problem to the tenths or hundredths place. Let's return to the problem 254 ÷ 8 and see how this works.

Language will be key here. It will be essential to talk in terms of tenths and hundredths to truly follow the place value, not in terms of "point 23" for .23. That number is "23 hundredths." Language matters in the development of place value.

Let's assume that the assignment is to divide through to the hundredths place. If that is the case, plan ahead and place the zeros into position in order to leave enough room.

8)254.00	30
-240.	
14.	1
- 8.	
6.0	.7
- 5.6	
.40	+ .05
- .40	31.75
0	

The Script:

- *10 8s would be 80 so 30 8s is 240 so that's closer*
- *254 minus 240 is 14*
- *One 8 is 8 which is the closest I can get*
- *14 minus 8 is 6, and no more whole groups of 8 are possible*
- *6 is the same as "60 tenths." I can get "56 tenths" out of 60 tenths, leaving 4 tenths leftover*
- *4 tenths is the same as 40 hundredths. 5 hundredths times 8 is 40 hundredths. I have no more left so my answer is 31 and 75 hundredths.*

Notice the need to flexibly think of 6 as 60 tenths. This is where your son's or daughter's understanding of how decimals and fractions are linked is important. When we do the American Standard Algorithm in Division we actually ignore the decimal point, bring a 0 down, and say, "How many 8s are in 60?" ***The shift is in emphasizing the mathematical idea of re-representing any number into an equivalent form. This is a core place value concept!***

Re-representing Numbers

When I ask you precisely how many 10s are in the number 254, the answer is *25.4 tens.* Precisely how many hundreds are in 3,254, the answer is 32.54 hundreds. The same number in terms of 10s is 325.4 tens. Thinking flexibly in this manner is a sign of deep place value understanding. We often do these conversions without going the extra step of labeling our answer. Consider the following example:

3.5 million Written as ones we would rewrite this as: 3,500,000 ones

How would you rewrite that same number as thousands? 3,500 thousands

The same skill is necessary when working with decimals. It is a question of *equivalent representations to make the numbers easier to work with!* $6 = \frac{60}{10} = \frac{600}{100}$ Remember back to our initial rules at the beginning of this series? Language matters in developing place value. If you don't like the numbers, change them, or in the case of decimals, re-represent them to make the numbers easier to work with. Always keep the mathematics visible.

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