Understanding Your Child's Mathematics

Addition Strategies, Part 2

Incremental & Compensation Strategies

Project for Elementary Mathematics
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Draft #7 2012

"Mental Math" Strategies: Incremental Strategy

Think of this as working 'piece-by-piece.' Some call this the 'Counting On' Strategy

This strategy has two variations. What is common to both is that a child breaks the number down into pieces then combines them one at a time. The difference is in how many pieces are created.

Variation #1: Keeping one number whole

$$\begin{array}{r} 86 \\ +\ 38 \\ \hline 116 \\ +\ \ 4 \\ \hline 120 \\ +\ \ 4 \\ \hline 124 \end{array}$$

Your Script:
86 + 30 is 116
116 + 4 (That's 4 of the 8) is 120
120 plus the other 4 of the 8 is 124

Stages of Learning: Examples of a child early use of this strategy might sound like the following:

- *86, (pause) 96, 106, 116, (pause), 117, 118, 119... 124 (Counts up by ones eight times.), or*
- *86, (pause) 96, 106, 116, (pause), plus 4 gets me to 120, plus another 4 gets me to 124.*

Variation #2: One piece at a time

$$\begin{array}{r} 86 \\ +\ 38 \\ \hline 110 \\ +\ \ 8 \\ \hline 118 \\ +\ \ 2 \\ \hline 120 \\ +\ \ 4 \\ \hline 124 \end{array}$$

Script difference:
Same as above. It is just that the numbers are broken down into place value parts first and then added piece by piece. *With time and practice, most of these small steps are quickly done mentally and speedily.*

Benefits:

1. Your child learns how to add a ten on to any number (86 + 10), not just from a round decade number like 80 + 10.
2. Your child learns the fact families of the single digit numbers (example: 8 = 4+4 = 5+3 = 6+2...)
3. They break these single digit numbers apart in order to quickly get to the next ten thus building fluency with organizing around ten.

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"Mental Math" Strategies: Compensation Strategies

Again, there are two variations. This time, however, the variations use two different ideas based on the concept of equality. The first strategy uses rounding to make the problem unequal to the original problem. The second version makes changes to both numbers to make sure everything stays equal at all times.

Version #1:

$$\begin{array}{r} 86 \\ +\ 38 \end{array} \quad \textit{round off to:} \quad \begin{array}{r} 86 \\ +\ 40 \\ \hline 126 \\ -\ \ 2 \\ \hline 124 \end{array}$$

Your Script:
I add 2 to 38 to make it 40 which is an easier number to work with.
86 + 40 = 126
Since I added 2 to 38, I have 2 too many in my answer so I need to subtract 2
126 - 2 = 124 My answer is 124.

Version #2

$$\begin{array}{r} 86 \\ +\ 38 \end{array} \quad \begin{array}{l} \textit{take 2 off of } 86 \\ \textit{and add it to } 38 \end{array} \quad \begin{array}{r} 84 \\ +\ 40 \\ \hline 124 \end{array}$$

Your Script:
To make both numbers easier to work with I take 2 off of the 86 and add it to the 38. This will give me the same answer as the original problem.
84 + 40 = 124

Important Mathematical Ideas:

To do either of these strategies well, a child needs to have a growing idea about what the 'equals' (=) sign means. Most children think = means the 'answer comes next.' This is not correct and severely limits his or her ability to work with important algebraic ideas later on. Variation #2 is built on the idea that I can keep the values equal even if I recombine the numbers in an equivalent amount. Example:

86 + 38 = (84 + 2) + 38 = 84 + (2 + 38) = 84 + 40

Version #1 requires a slightly different thinking about the issue of equality. The change made by rounding off just one number makes the problem become unequal (86 + 38 ≠ 86 + 40). The changed numbers, however, are a lot easier to work with. What a child needs to focus on is how does he or she undo the change. Since the one number was increased in size, the effect is the initial answer is too big so it needs to be made smaller by the amount originally added: 86 + 38 = 86 + (38 +2) - 2 = (86 + 40) - 2

There is a lot of algebraic ideas at work in both of these versions; ideas about equality being the most critical. But there is another skill being developed called 'relational thinking.' This skill allows a child to think about the effect of changes on a number and how it affects the outcome. Is it still of equal value? If I move this number to the other side of the equal sign to make the problem easier, what do I need to do to keep everything equal? As I said, we are getting into some algebra and elementary students are capable to grasping these ideas.

Limitations and Cautions:

• Don't limit your child by focusing on a 'rule' such as, *"If you add a number you subtract."* While true in addition, your child will run into problems when he or she gets to subtraction. Focus on the *effects* of the changes to the numbers, not the surface rule.

• The idea of this strategy is easy and makes sense to most children. Doing it initially is hard because how to readjust the numbers requires careful thinking. Use pictures or models that will help make the mathematics visible.

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