Understanding Your Child's Mathematics

Subtraction Strategies, Part 1

Standard Algorithm and Tens & Ones

Project for Elementary Mathematics
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We outlined three basic strategies in addition - Tens & Ones/Standard American Algorithm, Incremental, and Compensation. Each of these has a counterpart in subtraction with one extra strategy; that of adding up to find the difference between the two numbers. First, let's take a look at the strategy we were all taught in school and understand how it really works.

American Standard Algorithm in Subtraction

As with the addition standard algorithm, the struggle children have around learning it is not there is anything mathematical wrong with the way the algorithm works, the script we use to describe the steps of the procedure hides the mathematics. Let's review the procedure as we were taught it. We will use the same numbers as before, but this time subtract.

$$\begin{array}{r} {}^{7} \\ 8\,{}^{1}6 \\ -\ 3\,8 \\ \hline 4\,8 \end{array}$$

Classic Verbal Script:

- *I can't take 8 from 6 so I borrow 1 from the 8, make it a 7, put the 1 in front of the 6 .*
- *16 take away 8 is 8. Put down the 8.*
- *7 take away 3 is 4. Put down the 4. The answer is 48.*

This should sound familiar. At its core, *the script that is used,* and again I emphasize *the script,* asks you to ignore the *place value,* act upon the numbers *as a string of single digits*, and, if you place those digits in the correct location, you get the correct answer. Many children have been interviewed who honestly believe that all they have literally taken from the "eight" is a one, and the only reason the six becomes "sixteen" is because you placed the "one" in front of it. They do not understand mathematically how the 86 is altered. They do not recognize that the "^{7} and ^{1}6" is in reality 70 + 16 and that it is still worth 86. Remember the "rules" for helping your child? Let's look at the same algorithm but with *the place value language maintained, how the numbers were broken apart to make them easier, and the mathematics made visible.*

$$\begin{array}{rcl} 86 & = & 70 + 16 \\ -\ 38 & = & \underline{30 + \ \ 8} \\ 48 & = & 40 + \ \ 8 \end{array}$$

New Script:

- I don't have enough to take 8 from 6 so *I am going to break apart 86 into 70 and 16 to make 86 easier to work with. I also break 38 into 30 and 8.*
- *16 take away 8 is 8. 70 take away 30 is 40. 40 plus 8 is 48. The answer is 48.*

This script describes why and how 86 is broken apart. Breaking apart numbers is a very important idea and skill for children to grasp. (See Rule 3: If you don't like the numbers...) Expanding the math off to the side helps make the numerical changes more visible. I don't expect children to go into adulthood having to do this expanded version, but I do anticipate them having to stay in this stage for a while to solidify their place value. They can eventually learn the more collapsed version with its shortcut notations. The difference, however, is that his or her mental script will reflect the legitimate changes to the numbers and maintain place value at all times.

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"Mental Math" Strategies: Tens & Ones Strategy

Also known as 'Partial Differences'

I have paired the 'Tens & Ones Strategy' with the 'American Standard Algorithm in Subtraction' because both use similar place value concepts. The Standard Algorithm makes changes to the "top" number in order to then subtract the *ones* and then subtract *the tens.* Let's see what happens if a child tries and starts with the tens first as can easily be done in addition.

$$\begin{array}{r} 86 \\ -\ 38 \\ \hline 50 \end{array}$$

Script:
80 minus 30 is 50.

So far, so good! But now, there is a *potential problem.* I say potential because it's only a problem if you are limited to thinking "you can't take 8 from 6." (See Rule 2: Don't Place Unnecessary Limits on Your Child.) There are two ways of thinking about the situation of 6 - 8: **one** is, if you know that *zero can be bridged,* meaning there are numbers on the other side of zero, your child can use negative numbers. **The second is asking the question, "***what am I short?"* Let's continue with each scenario one at a time.

Scenario 1: *"Going Negative"*

$$\begin{array}{r} 86 \\ -\ 38 \\ \hline 50 \\ -\ \ 2 \\ \hline 48 \end{array}$$

Script:
- *80 minus 30 is 50.*
- *6 minus 8 is negative 2 (⁻2)*
- *50 and ⁻2 is 48. The answer is 48.*

Note that technically your child is adding 50 + ⁻2 but that has the same effect as 50 - 2. This is a long term association involving *inverse elements,* but I would let that sit in the background for now and let your young child use the equivalent positive expression.

Scenario 2: *"What am I short?"*

$$\begin{array}{r} 86 \\ -\ 38 \\ \hline 50 \\ -\ \ 2 \\ \hline 48 \end{array}$$

Script:
- *80 minus 30 is 50.*
- *6 minus 8, I can take away 6 but I'm short 2, so the 2 has to come out of the 50. 50 minus 2 is 48. The answer is 48.*

So, if your child is not comfortable thinking about negative numbers, your child is usually very aware what he or she is short of if they have six and want eight.

Benefits:

Using the Tens & Ones Strategy in subtraction maintains place values at all times. Using it, however, does compel your child to think about other mathematics different than the Standard Algorithm. Your child needs to either think about how zero can be crossed over into the realm of negative numbers, or your child needs to understand the difference between two numbers, do you have enough or are you short an amount (in debt so to speak). With this last idea, if I am short an amount, I need to take it out of what I have left. In this case it has to come out of the remaining fifty. Saying 6 - 8 can't be done sets up huge misconceptions for later work with algebra.

The strict use of the Standard Algorithm typically would say that 50 - 2 cannot be done because you can't take 2 from zero. But that is not efficient! I want all children to look at 50 as a whole number and be able to know what two less would be. That is a reasonable expectation!

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