

Understanding Your Child's Mathematics

Subtraction Strategies, Part 2

Incremental and Compensation Strategies

Project for Elementary Mathematics
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"Mental Math" Strategies: Incremental

Also described as working "Piece-by-Piece"

This strategy has some advantages in subtraction over the Tens & Ones approach. Finding the subtraction of six minus eight (6 - 8) as too confusing from a Tens & Ones perspective, some children begin to work with each "piece" of the number one at a time. The big deal in subtraction is how do you go from one decade to the decade below. Example: In 24 - 6, the mathematical idea is how do you get from the twenties to the teens without necessarily breaking the 24 into 10 + 14 as you do in the American Standard Algorithm. You have some choices. Here are two typical ways children negotiate this.

Version #1a & 1b:

a) $\begin{array}{r} 86 \\ - 38 \\ \hline 50 \\ - 8 \\ \hline 42 \\ + 6 \\ \hline 48 \end{array}$	b) $\begin{array}{r} 86 \\ - 38 \\ \hline 50 \\ + 6 \\ \hline 56 \\ - 6 \\ \hline 50 \\ - 2 \\ \hline 48 \end{array}$
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Script:

- 80 minus 30 is 50.
- Now you have two choices, deal with the 8 first or deal with the 6 first.
- a) The 8 first: 50 - 8 is 42. I have add the six back on so 42 + 6 is 48. The answer is 48.
- b) Now the 6 first: 50+6 is 56. 8 = 6+2 so 56 - 6 is 50, minus 2 is 48. The answer is 48.

What's in Common? A child who works this way is looking at 86 and 38 by their place value components. The child also is exploring the idea that the order of the operations can be altered. There is no limitation here in that regard. But instead of breaking 86 into 70 + 16 as in the standard algorithm, the child is breaking the 38 into 30 + 6 + 2 (version 1b). The adage of, "If you don't like the numbers you have, change them!" is being used here just as in the Standard Algorithm, it's just that the numbers be altered is the bottom number not the top number. Both choices work!

Version #2:

$$\begin{array}{r} 86 \\ - 38 \\ \hline 56 \\ - 6 \\ \hline 50 \\ - 2 \\ \hline 48 \end{array}$$

Script:

- 86 minus 30 is 56.
- 8 = 6+2 so I will subtract it in two parts, 56 - 6 is 50 (that's 6 of the 8), minus the 2 (the last of the 8) is 48. The answer is 48.

Here the child keeps the larger number whole and took of the tens and then the ones of the bottom number. This saves time compared to the first versions, but the mathematical ideas are the same.

Early Versions: When children first try these strategies, they may need to count back one ten at a time (86, 76, 66, 56, (pause) 55, 54, 53...48). This is normal! As their number sense matures, and they are pushed to think about saving themselves time, they come to work in bigger and bigger chunks of numbers and become very fast.

"Mental Math" Strategies: Compensation

This is the strategy where rounding numbers to make them easier and quicker to work with becomes the skill to be cultivated. Like in addition, your child needs to keep track of how the relationships between the numbers are changed if just one of the numbers is changed or if both numbers are changed. You really cannot get by with simple rules here. Subtraction distorts the changes differently than in addition. Let's see how this works with the numbers we have been using.

Version #1:

$$\begin{array}{r} 86 \\ - 38 \\ \hline \end{array} \text{ round off to: } \begin{array}{r} 86 \\ - 40 \\ \hline 46 \\ + 2 \\ \hline 48 \end{array}$$

Script:

- *I round off 38 to 40 to make the problem easier to work with.*
- *86 minus 40 is 46.*
- *But with 40 I took 2 too many so I need to add 2 back on so 46 plus 2 is 48. The answer is 48.*

Notice how this is different than in addition. By making the number larger, more was taken away. If you look at this on a number line, by adding 2 you *shorten the difference* between the two numbers, so you need to add the two back on to lengthen the distance to the original.

Version #2

$$\begin{array}{r} 86 \\ - 38 \\ \hline \end{array} \text{ add 2 to 86 } \begin{array}{r} 88 \\ - 40 \\ \hline 48 \end{array} \text{ add 2 to 38}$$

Script:

- *If I add 2 to both numbers I get 88 - 40 which will have the same difference as 86 - 38. 88 minus 40 is 48. The answer is 48.*

Notice that this compensation strategy is solely based upon the *difference between* two numbers rather than the *take away* model with which we are more accustomed. $86 - 38 = 88 - 40$. Look at this on a number line to see why this works all the time. Important ideas of equivalency are being developed here. This strategy certainly takes advantage of the rule, "If you don't like the numbers you have to work with, change them!"

There is another version, less used, but worth noting. Paying attention to it re-enforces the caution about not focusing on memory rules but instead focusing on how the relationship between the numbers are affected. The two above versions were based upon changing either just the lower or both numbers. What would happen if we were to just change the top number?

Version #3:

$$\begin{array}{r} 86 \\ - 38 \\ \hline \end{array} \text{ round off to: } \begin{array}{r} 90 \\ - 38 \\ \hline 60 \\ - 8 \\ \hline 52 \\ - 4 \\ \hline 48 \end{array}$$

Your Script:

- *I add 4 to 86 to make it easier to work with. 90 minus 30 is 60. 60 minus 8 is 52. But by making 86 ninety, I have 4 too many so I need to take that 4 off so 52 minus 2 is 50 minus another 2 is 48. (Notice the mix of strategy ideas from other those described on other pages)*

Notice here the similarity with the addition compensation decisions. The adjustments are different than when the bottom number is rounded off. Simple memory rules such as "if I add it I have to subtract it," can trap you into making an error. It's better to track the changes in the relations than rely on simple surface rules.