Understanding Your Child's Mathematics Subtraction Strategies, Part 3 Adding Up: The Inverse Operation

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Adding Up: "The Difference Between" (Or think of this as the "Giving Back Change" Strategy)

Consider the following problem, *"TJ earned \$400 over the summer mowing lawns for families in the neighborhood. At the end of the summer, he bought a bicycle for \$294. How much money does TJ leftover?"* Now, the context of this problem is a subtraction, "take away" structure. But let me write it out in a column format.



Yes, I could do the American Standard Algorithm for Subtraction by `changing the 400 into 300 + 90 + 10 and then subtract each part of the 294 away from each of the top number parts. I could use the Incremental or Tens & Ones strategies. But given these numbers many individuals will naturally add up. They change the problem into \$294 + ____ = \$400. The script would sound like the following: "294 + 6 gets me to 300. 300 plus another 100 gets me to 400. The answer is \$106."

Two important mathematical ideas are critical here. The first is that *any* subtraction problem can be restructured into an addition problem. This is a fairly sophisticated abstract idea. Where most children see adults use this strategy is in making change. The "School" version of making change is to use subtraction, but that is not that case in the world of commerce. You add up to make change. Once children realize this, doors are open to think more flexibly about numbers.

"Take Away" and the "Difference Between"

The second mathematical idea that is important for your child to develop is that there are two ways to think about a subtraction problem, one is the very familiar "Take Away" strategy; the other is the less familiar "Difference Between" strategy. Consider this problem, ""The score of the basketball game was 63 to 47. By how much did the winning team beat the losing team?" Most of us were trained in school to think of this as a "Take Away" problem - "63 take away 47 is..." - and write it out as 63 - 47. But if I think of this as the difference between two numbers like the comparison context of the problem implies, then I could solve it as 63 - $__$ = 47, or I could turn the problem into an addition problem and solve it as 47 + $__$ = 63. So, let's practice these three strategies. Each of the two subtraction approaches has been described earlier.

American Standard	Difference Between	Difference Between
Algorithm	Strategy	Strategy
(made visible)	(Counting back: Incremental)	(Adding Up: Incremental)
63 = 50 + 13 - 47 = 40 + 7 16 = 10 + 6	$63 - \underline{10} \rightarrow 53 - \underline{3} \rightarrow 50 - \underline{3}$ $\rightarrow 47; \ 10 + 3 + 3 = 16$	$47 + \underline{3} \rightarrow 50 + \underline{13} \rightarrow 63$ $13 + 3 = 16$

Which one is better?

As they say around here, "It all depends on if you are from Minneapolis or St. Paul!" meaning you are both looking at the Mississippi River but each from your own side. Some of this is preferred choice. However, sometimes there is a legitimate case for one being more efficient than the other. In the case of \$400 - 296, I would argue that either of the Difference Between Strategies are quicker and more efficient. (We will talk about efficiency soon.) The numbers 63 - 47 are so close together that looking at these numbers in terms of their difference is arguably more efficient. The important mathematical idea, however, is that your child needs to understand both the Take Away and the Difference Between strategies. To make an efficient choice, you need to be fluent with all three strategies.

But what about really big numbers? Does Adding Up work there?

Let's practice using some classically very unfriendly number combinations. I present the problem as a subtraction problem as it would potentially appear on a test or worksheet. The addition components are written off to the side as you might jot them down to keep track of your thinking.

If you choose to use the Adding Up: Difference Between Strategy, it can be very fast and very efficient. Notice, however, in the script that to become good at it, you have to be good at moving in chunks of numbers using place value skills and getting to landmark numbers. These skills need to be consciously built.