Understanding Your Child's Mathematics
Addition & Subtraction Summary
Why Bother? What's the Advantage?

Project for Elementary Mathematics
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Let's Review:

The Four Rules to Help Your Child:

1. Watch how you talk about the numbers.
2. Don't place unnecessary limits on your child.
3. If you don't like the numbers you have to work with, change them!
4. Make the math visible.

The Mathematics

Place Value: Without it, numbers are just a pile of ones! And as numbers get bigger, counting everything by ones gets a little tedious. Place value matures over time, in fact, deep elements of it do not fully mature until a child is in fifth grade. Place value is more than just saying that the "three" in 4,327 is worth 300. But that "three" is also 300 ones, 30 x 10, 3 x 100, as well as multiple other sub-parts such as 200 + 100, or 100 + 100 + 100. Place value is about multiplicative relationships. How a number is built (20 + 3, or 20 + __ = 27), how a number is broken apart (17 – 7), and how a number is compared (30 compared to 34, 23 compared to 33), are among the different ways numbers need to be thought about where elements of place value are used. Without a strong, deep sense of place value, your child is sunk! This is one of the reasons why it is important to describe *the value of the numbers* when adding or subtracting rather than calling them *by their digit names.* Keep the place value the focus. Shortcuts that ignore place value do more long range damage than short-term gain.

Breaking numbers apart: That 10 is made up of 7 + 3 or 6 + 4 or any of its other sub-parts is important for a child to understand. If it's not easy taking 18 away from 45, I need to know that I can break down the 45 into something easier (typically 30 + 15) or I can break down the 18 (typically 10 + 5 + 3). So much of what happens in mathematics is knowing different ways numbers can be broken down into smaller, easier to work with combinations.

Multiple Strategies: Each of the different addition and subtraction strategies draws on some different mathematical skills. Taking 84 - 30 (subtracting a group of ten from any number) is a slightly different skill than taking the 30 from the 80 and then adding the 4 back on. The Tens & Ones (84 - 36, 80 - 30 = 50, 4 - 6 = $^-$2) strategy in subtraction causes an individual to either think about negative numbers (and that there is life on the other side of zero!) or think about how I am "short 2" and that that 2 needs to be taken out of the remaining 50. By being fluent in several strategies, a child learns a broader range of number skills. He or she thinks about numbers from many different angles instead of just those ideas around one single strategy. The child becomes more flexible working with the numbers and can make many more judgments about how to work the numbers. By not putting artificial limitations on children ("Such as, *"You can't take 6 from 4."*) doors are left open to be explored when the time is ripe.

The Math, The Algebra: When you go to add 54 and 38, it does not mathematically matter whether you add the ones first or the tens first or only one group of ten to the other whole number. First, you are breaking the numbers into place value components. Second, you are using the commutative property of addition to move the numbers around. So in this one instance, the place value was maintained, there were no limits placed on the order of the addition and the math was kept visible. An algebraic property, the commutative property, was used. Having your child explore the commutative property is important for two reasons: first, it can make chunking combinations of numbers together easier, and, two, it has its limits. 2 + 18 may equal 18 + 2 but 18 - 2 does not equal 2 - 18. There is a *mathematical* limit that *is necessary* to explore and comprehend. The order of operations may *sometimes* (there are definite limits here) be tinkered with. The Incremental Subtraction Strategy is one such place where this can happen. Recall, 86 - 38. I can break this down into 80 - 30 + 6 - 8, but just as easily I could do 80 - 30 - 8 + 6. Or, I could even, if it was really worth it, do 80 - 8 - 30 + 6. I will get the same answer each time. But be careful, it does not work with all mixes of operations and that is why parentheses ( ) were invented!

When we looked at the compensation addition strategies where we did some rounding of numbers to make the addition easier, issues of equality were important. The number 86 + 49 may be easier to add if you think of it at first as 86 + 50, but 86 + 49 does not equal 86 + 50. I need to subtract the one number I added on to bring the numbers back into equilibrium. On the other hand, if I am good at breaking numbers apart into equal sub-parts, I can redistribute the values of the numbers to make them easier to work with. 86 + 49 can be broken down into 85 + 1 + 49. The 1 can then be added onto the 49 to make it 50. 86 + 49 does equal 85 + 50. To do that I need to understand about decomposing numbers, the associative property in addition (I can added neighboring numbers in any order), and I maintained equivalent relationships. All of this uses elements of algebra.

We will talk more about the algebra underlying the arithmetic one does see in a later section.

The Advantage:

Your son or daughter will be a very different everyday mathematician than you or I if they learn and focus on the variety of strategies described so far. Your child's place value will mature earlier and more soundly. His or her ability to make judgments about numbers and be flexible will be stronger. If the underlying ideas of the mathematics are made visible, your child's algebraic thinking will be more visible. You will hear the difference as they work and talk about the numbers. They will be ready for the workplace as they get older because they will be able to do and understand more. And in our economy, that's an advantage!

But what about multiplication and division? What's expected there? Let's take a look.

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