Creating Number Strings: Instructional Tasks to Foster Relational Thinking & Computational Fluency

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Number Strings

Number strings – also referred in some materials as *Mental Math Strings* (Fosnot & Dolk, 2001), *Problem Stems* (Carpenter, et al., 1999), or *series of equations* (Carpenter, et al., 2003) – are instructional tasks designed to have students complete the first expression or equation and then use the information in that task to complete the next one. Example: If you know 6 + 6 = 12, can you use that information to help figure out 6 + 7? The object is to have students recognize that since 7 is one more than 6, then the answer to 6 + 7 is one more than 6 + 6. This helps students gain the capacity to think relationally across tasks.

Such strings are typically organized around a specific mathematical intent. The strings systematically begin from an accessible entry point of all or most students in the class. With each successive item that follows, the intent is to increase the sophistication of either the numbers involved or deepen the conversation around the mathematical concept being explored. Consider the following example:

Mathematical Intent: Developing the make a ten strategy in addition

7 + 3	*What is 7 + 3?* [10]
	How can you use what you know about 7 + 3 to figure out what is…
7 + 5	

The string can continue by circulating back to other combinations of ten such as 8 + 2, followed by 8 + 6. In this process, students develop an understanding that 6 can be broken into a variety of combinations. Context drives which combination of 6 is desirable. In the case of 8 + 6, and if *making a ten* is the strategy objective, decomposing 6 into 2 + 4 allows the 2 to be added to 8 to make 10 leaving 10 + 4 to be determined. Thus:

$$8 + 6 = 8 + (2 + 4) = (8 + 2) + 4 = 10 + 4 = 14$$

Number stings need not be limited to math expressions such at 7 + 3 followed by 7 + 5. Strings can include instructional formats such as true/false equations, open number sentences, or a combination of them all. These instructional tasks will be described in an upcoming section.

The following sections reflect upon the design features involved in crafting and presenting such number strings. These sections will explore establishing:

- Mathematical Intent
- Number choices
- Instructional formats, and
- Representational choices

Mathematical Intent

The mathematical intent, or learning objective of the string, influences the number choices both within an expression or equation to then how the numbers progress from the first to the second, third, etc. combination in the sequence. (See figure 1) The mathematics to be cultivated can develop a particular strategy, such as the *Make a Ten* strategy explained previously or an algebraic property. For instance, the *Make a Ten* strategy also involved the use of the *associative property of addition.* In the equation chain of…

$$8 + 6 = 8 + (2 + 4) = (8 + 2) + 4 = 10 + 4 = 14$$

…the 2 was re-associated from the 6 and added to the 8 to make an easier combination of 10 + 4. This is a fundamental algebraic property based upon the idea that a number is made up of sub-units that can be recombined to from an equivalent combination.

The associative property also applies to the operation of multiplication. One fact combination that students take longer to recall with automaticity is 8 x 6. A number string can be devised to help students explore these relationships. Example:

4 x 6	*What is 4 x 6?* [24]
8 x 6	*Can you use 4 x 6 to figure out 8 x 6?*

Assuming that students have 4 x 6 at a recall level, at least some students within the group will notice that 8 is double 4 and as a result will double 24. This scale factor relationship, however, may be implicitly understood among students but may remain out of conscious use if only left as a verbal explanation. How the associative property is explicitly exposed becomes a key instructional decision on the part of the teacher. The teacher at this point needs to *make the math visible* to students by deciding how to represent the student's explanation.

Making the Math Visible – Mathematical Representations of Student Explanations

Consider the following two options for representing the number string example of 4 x 6 and 8 x 6:

4 x 6 = 24
8 x 6 = 48

This representation creates what can be referred to as a *measure space.* The scale factor relation between the multipliers of 4 and 8, and the factor relation between the products of 24 and 48 are both 2. The teacher can capture this relationship visually as follows:

4 x 6 = 24
x 2 8 x 6 = 48 x2

Adding arrows and adding the factor of x2 to each side draws attention to scalar relation. It does not, however, draw attention to the associative property. Consider the following:

4 x 6
8 x 6 = (2 x 4) x 6
= 2 x (4 x 6)

So you doubled 4 x 6 to figure out 8 x 6? Look at the numbers on both sides of the equal sign. Is there another way to see why 8 x 6 is the same as 2 times 4 time 6?

This representational format draws out that 8 can be factored into 2 x 4 and the 4 can be re-associated with the 6 in order to use a known combination to figure out an unknown combination. The prompt from the teacher draws attention mathematical decision. The implicit becomes explicit.

Instructional Tasks

Each string present thus far has initially been a series of mathematical expressions, e.g., 7 + 3. Equations are two mathematical expressions compared across the equal sign, e.g., 8 x 6 = 2 x 4 x 6. Equations highlighting a mathematical relation and/or property can also be used in such strings. Strings can be initiated using other instructional tasks such as true/false equations, open number sentences or in combination. Consider the following string:

9 + 6 = 10 + 6	True or false? Why?
9 + 6 = 9 + 1 + 6	True or false? Why?
9 + 4 = 10 + ☐	What goes in the box to make it true?
7 + 8 = 7 + ☐+ 5	What goes in the box to make it true?

The mathematical intent of this number string was to explore both the compensation strategy of rounding a 9 to make it 10 – a much easier combination with which to work – then extending to the strategy of Make a Ten [7 + 8] to see the interconnections. Both true-false combinations as well as open number sentences allow students to engage in considering the mathematical relations and properties. An understanding of equality, decomposition of number, and the associative property also underlay the mathematical strategy.

Being Flexible

Being flexible is important when enacting any string. You may have a mathematical intent in mind but that does not mean that a student will see and articulate a legitimate alternative strategy. Responding to the mathematical integrity of ideas is more important.

Summary

To design strings such as these, the following needs to be considered:

- Clarity of the mathematical intent
- Selecting an initial, highly accessible number choice as a starting point to engage the largest number of students
- Instructional tasks to use: expressions, equations, true/false statements, open number sentences
- Deciding on the representational format to use to make the mathematical ideas publicly visible
- The questions and prompts to help students to reflect upon their actions and consider the deeper mathematical understandings

References

• Carpenter, T. P., E. Fennema, M.L. Franke, L. Levi, S.B. Empson (1999). *Children's Mathematics: Cognitively Guided Instruction.* Portsmouth, NH: Heinemann.

• Carpenter, T. P., M.L. Franke, L. Levi (2003) *Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School.* Portsmouth, NH: Heinemann Press.

• Fosnot, C.T., M. Dolk. (2001). *Young Mathematicians at Work: Constructing Multiplication and Division.* Portsmouth, NH: Heinemann.

Figure One

Mathematical Intent – Concepts to Develop

Equality [=]

4 x 7 = 2 x 7 + 2 x 7 True or false?
13 = 13

Computational Strategies – Addition & Subtraction:

- Make a Ten: 8 + 5 = 8 + 2 + 5; 26 + 6 = 30 + 2
- Get Back to 10: 13 – 5 = 13 – 3 – 2; 64 – 7 = 64 – 4 – 3
- Doubles ± 1: 8 + 7 = 8 + 8 – 1; 8 + 7 = 7 + 7 + 1
- Compensation: 29 + 7 = 30 + 7 – 1

Algebraic Properties

- Associative Property (addition & multiplication)
  8 + 5 = 8 + (2 + 3) = (8 + 2) + 3
  8 x 5 = (3 x 5) x 5 = 3 x (5 x 5)
- Distributive Property (multiplication):
  8 x 7 = (3 + 5) x 7 = (3 x 7) + (5 x 7)
- Commutative Properties (addition & multiplication)
  4 + 25 = 25 + 4
  25 x 4 = 4 x 25 [25 groups of 4 has the same number of total items as 4 groups of 25]
- Identity Properties
  a + 0 a – 0
  a x 1 a ÷ 1

Try These

Addition:	#1	6 + 4 6 + 5 7 + 3 7 + 5	#2	6 + 7 = 7 + 7 + 1 T/f? 6 + 7 = 6 + 6 + ☐ 8 + ☐ = 7 + 7 + 1 29 + 29 = 30 + 30 – ☐
Subtraction:	#1	10 – 3 13 – 3 13 – 6	#2	7 + 9 = 6 + 10 T/F? 13 – 9 = 12 – 10 T/F? 13 – 9 = ☐ - 10
Multiplication:	#1	3 x 6 6 x 6 9 x 6 10 x 6	#2	